Labor-Market Hysteresis and Persistent Paradox-of-Thrift Recessions*

Tobias Broer† Jeppe Druedahl‡ Karl Harmenberg§ Erik Öberg¶

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Abstract

Following the recent disruption in production due to COVID-19, we investigate whether temporary adverse shocks can result in persistent demand-driven recessions through sluggish labor-market dynamics. We consider an incomplete-markets model with sticky prices and search frictions, and show how introducing sluggish vacancy creation and endogenous layoffs gives rise to a powerful and persistent feedback loop between unemployment risk and aggregate demand. Endogenous layoffs are central because they generate a rapid rise in unemployment following a temporary shock. Sluggish vacancy creation is central because it implies that job-finding rates remain persistently low following the surge in layoffs. As a result, the negative feedback loop continues even after the initial shock dies out. The feedback mechanism is weak in the corner cases of either free entry, exogenous separations or complete markets. The model provides justification for using match-saving subsidies to stabilize the business cycle.

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†IIES, Paris School of Economics and CEPR. tobias.broer@iies.su.se.

‡University of Copenhagen and CEBI. jeppe.duredahl@econ.ku.dk.

§Copenhagen Business School. kha.eco@cbs.dk.

¶Uppsala University and UCLS. erik.oberg@nek.uu.se.

1 Introduction

Several recent US recessions have seen a persistent deterioration of labor-market conditions and aggregate activity after seemingly temporary shocks.[1] This explains, perhaps, the widespread worry that the current economic disruption from COVID-19 may lead to a similarly prolonged recession. In particular, policymakers and academics alike have expressed a concern that the sharp increase in job losses currently seen in many countries may be amplified by a strong fall in demand, turning a temporary disruption into a deep, prolonged recession.[2] In this paper, we investigate under what conditions this worry is justified, and which stabilization policies it may warrant.

To this aim, we present a heterogeneous-agent new-Keynesian (HANK) model with search-and-matching (SAM) frictions. As is standard in the growing HANK literature, prices are sticky and asset markets are incomplete. As is standard in the SAM literature, matching is random. To this environment we add two features: endogenous separations due to firm-specific stochastic continuation costs and sluggish vacancy creation due to firm-specific stochastic entry costs. Our main contribution is showing how these features interact to give a powerful demand-driven propagation mechanism, by which a temporary shock to aggregate productivity leads to a surge in layoffs followed by a persistent decline in job-finding rates. The result is large and persistent fall in consumption, inflation and aggregate activity. Importantly, in alternative versions of our model where either one of these elements is not operative we find substantially smaller propagation. Because the propagation mechanism relies on the interaction of the two features, policies that undo any one of them can stabilize the recession. In particular, governments can close the output gap by subsidizing firms to keep existing matches alive.

To understand the mechanism, consider an initial and temporary fall in productivity. The direct effect of the fall in productivity is an increase in layoffs from the endogenous job

[1] See Hall and Kudlyak (2020). For a discussion of the slow recovery following the US recession in the early eighties, see Summers (1986); following the US recession in the early nineties, see Gordon (1993) and following the financial crisis in 2008, see Eeckhout and Lindenlaub (2019). The high persistence of European unemployment in the eighties is discussed in Blanchard and Summers (1986).

[2] See, e.g., Furman (2020), Portes (2020), Sahm (2020) and Powell (2020).

separations, leading to an increase in unemployment. Because vacancy creation is sluggish, the surge in unemployment following the layoffs implies that labor-market tightness remains depressed beyond the temporary fall in productivity, with persistently high unemployment and low job-finding rates as a result. The fall in the job-finding rate leads to an increased precautionary saving motive for the workers, and with sticky prices, this leads to a persistent 'paradox-of-thrift' fall in aggregate demand. The fall in aggregate demand further amplifies the rise in layoffs and prolongs the fall in the job-finding rate, which together deepen the persistent fall in employment and aggregate activity.

Contrast our framework with a standard SAM model where job matches separate at an exogenous fixed rate and firms can freely create vacancies by paying a flow cost. First, because of the exogenous job separations, there is no direct increase in layoffs, and therefore a smaller initial increase in unemployment. Second, free entry implies that vacancy creation is infinitely elastic, implying that vacancy creation and tightness are jump variables that adjust to keep expected profits from vacancy posting at zero. In this standard environment, a temporary decline in productivity leads to a decline in tightness (and therefore job-finding rates) as well as a (small) rise in unemployment. But tightness returns to steady state as soon as the underlying shock has reverted. With job-finding rates as high as they are in US data, any effect of the shock on unemployment subsides quickly thereafter. Therefore, even when workers cannot insure against unemployment risk, a temporary shock does not raise the precautionary saving motive beyond the duration of the shock since the probability of job loss is exogenous and constant, and expected job-finding rates fall only temporarily.

The feedback in our model arises from the interaction of i) sticky prices, ii) incomplete markets, iii) endogenous separations and iv) sluggish vacancy creation. In particular, when we turn any of these elements off, the large and persistent demand-driven deviations of unemployment, inflation and output from their flexible-price levels disappears. It is thus the joint interaction of these ingredients that gives rise to the strong internal feedback mechanism. This has immediate implications for policy design: if governments can undo any one of these elements, they can effectively stabilize the output gap. In particular, we show that one way of achieving this is to prevent the endogenous rise in job destruction by subsidizing existing matches.

Our framework merges three existing models in the literature. The basic structure of our framework is similar to the HANK-SAM models by Ravn and Sterk (2020), Den Haan et al. (2018) and McKay and Reis (2020). In particular, the basic amplification mechanism in our paper, in which unemployment risk and precautionary savings endogenously feed each other to produce a demand-driven decline in output following a productivity shock, is similar to the mechanism in these papers. To this, we add endogenous separations similar to Mortensen and Pissarides (1994) and sluggish vacancy creation following Coles and Kelishomi (2018).[3] By introducing stochastic entry and continuation costs, we introduce an elasticity of job destruction to the value of a job and an elasticity of vacancy creation to the value of a vacancy. The standard SAM model is the special case with a zero elasticity of job destruction and an infinite elasticity of vacancy creation. Following Krusell et al. (2011), we study the model in the zero-liquidity limit, such that the model admits analytical aggregation, and can easily be solved by standard linearization techniques (e.g. by using Dynare).

Our analysis highlights the benefits of countercyclical labor-market policies for stabilizing demand. We do not, however, attempt to derive an "optimal" policy, which would require a more careful modeling of labor-market hysteresis. The source of hysteresis in our environment, a firm-specific stochastic entry cost, is a simple way to make the standard model consistent with a slow recovery in job-finding rates. There are several other, potentially important features that might give rise to similar phenomena, e.g., skill loss upon unemployment, destruction of match capital as well as firm credit constraints and defaults. While we believe that our simple mechanism captures the consequences of these phenomena for workers and their consumption-saving decisions in a tractable way, it would be fruitful to include these other margins explicitly in a framework similar to ours to analyze the costs and benefits of government policies in more detail.

[3] Coles and Kelishomi (2018) show, with an estimated process for labor productivity and separations for the US, that their model can account for several puzzles that a standard calibration of the free-entry model cannot: i) unemployment volatility, ii) autocorrelation in tightness and iii) correlation of unemployment and vacancies in the data.

Several features of our model are consistent with past recession experiences and current commentary about the development of the COVID-19 crisis. Standard new-Keynesian models predict a demand-driven output-gap expansion following a negative supply shock (see, e.g., Galí (1999)), whereas our model predicts a demand-driven recession, providing a rationale for much of the policy actions that have been taken across several countries since the outbreak of COVID-19. However, it is not central for our analysis that the initial shock is to supply; a negative demand shock will set off the same type of internal propagation mechanism.[4] The evolution of the recession-recovery path following a temporary supply shock in our model is what Reis (2020) dubbed an "ABC recession", by which an initial shock produces a large negative spike in output, followed by a partial rebound after the shock has reverted, and then a slow recovery afterwards. In our model, the initial deterioration in unemployment is driven by a spike in separations, which is followed by slow recovery in job-finding rates, broadly consistent with past US recession experiences (Hall and Kudlyak, 2020).

Our paper adds to the growing literature on the macroeconomic effects of COVID-19. One strand of the literature has focused on the interaction between the disease-spread, social distancing and economic outcomes, see, e.g., Eichenbaum et al. (2020) and references therein. Another strand has either interpreted COVID-19 as a shock to productivity or the demand for certain goods, and studied the macroeconomic consequences thereof, see, e.g., Guerrieri et al. (2020) and Faria-e Castro (2020). With respect to the labor-market effects of COVID-19, another closely related paper is Gregory et al. (2020), who similarly study the persistence of labor-market dynamics using a SAM model, but with a focus on the role of on-the-job search. For us, COVID-19 is a particularly pressing motivation to investigate the demand-propagation of temporary shocks, but we believe that the mechanism that we highlight is relevant for understanding business-cycle dynamics from a more general viewpoint.

More broadly, our paper adds to the now large literature on business cycles using HANK

[4] Brinca et al. (2020) provide evidence that the COVID-19 shock may be regarded as a simultaneous negative shock to demand and supply.

models and unemployment dynamics using SAM models.[5] A closely related paper is Challe (2020), which points out that with countercyclical income risk, supply shocks can lead to demand shortages through household's precautionary savings motive. Our contribution is to highlight the particular powerful interaction between incomplete markets and search frictions that arises when separations are endogenous and vacancy creation is sluggish, and which together can produce a deep and prolonged demand-driven recession.

On policy, our paper relates to the growing literature studying match-saving firm subsidies (e.g., furlough policies). Most of the literature has focused on these policies from a micro perspective, see, e.g., Guipponi and Landais (2018), Cahuc et al. (2018) and Bennedsen et al. (2020). Our contribution is to highlight the macroeconomic stabilization effect of such policies.

The paper is organized as follows. Section 2 presents our model. Section 3 presents a graphical framework for analyzing the model interactions. We calibrate the model in Section 4 and analyze the response to a short-lived TFP shock, and the stabilizing effect of match-saving subsidies, in Section 5. Section 6 concludes.

2 Model

Our model combines three canonical frameworks in the business-cycle literature. First, the production side has a basic new-Keynesian structure, with monopolistic competition and sticky prices. Second, to produce, firms must employ workers in a frictional labor market, following a basic Diamond-Mortensen-Pissarides setup. Finally, households cannot insure the resulting unemployment risk, but only invest in a risk-free asset, following a basic Aiyagari (1994)-Huggett (1993) setup. In so doing, our framework is similar to Ravn and Sterk (2020), and our model admits analytical aggregation through a no-borrowing constraint and a zero net supply of assets.[6] Distinguishing our model is that we will assume that separations are

[5]For a recent overview of the HANK literature, see Kaplan and Violante (2018). For a recent overview of the unemployment-dynamics literature, see Ljungqvist and Sargent (2017).

[6]The convenience assumptions of no borrowing and zero net asset supply was used in the context of asset pricing by Krusell et al. (2011) and has been used extensively in the HANK literature since, see, e.g.,

endogenous, following Mortensen and Pissarides (1994), and that vacancy creation is sluggish because firms must pay a stochastic entry cost, following Coles and Kelishomi (2018).

Specifically, the economy consists of infinitely-lived workers indexed by $i \in [0,1]$, and infinitely-lived capitalists indexed by $i \in (1, 1+\text{pop}_c]$. The workers have CRRA preferences with discount factor β and risk aversion σ. The capitalists are risk neutral with discount factor β and own all firms. Production has three layers:

1. Intermediate-good producers hire labor in a frictional labor market with search and matching, producing a homogeneous good sold in a perfectly competitive market.

2. Wholesale firms buy intermediate goods and produce differentiated goods sold in a market with monopolistic competition. The wholesale firms set their prices subject to a Rotemberg adjustment cost.

3. Final-good firms buy goods from wholesale firms and bundle them in a final good, which is sold in a perfectly competitive market.

We first describe the within-period timing in the model, then the determination of vacancy posting and job separations in the frictional labor market, then the price-setting mechanism in the wholesale and final goods market, and finally the households' consumption-saving decisions.

2.1 Timing and labor-market dynamics

Step 0: Stocks and productivity. In the beginning of each period t, aggregate productivity A_t is revealed. The state variables are the stock of unemployed workers $\underline{u}_t$, and the stock of existing vacancies $\underline{v}_t$.

Step 1: Entry of firms. Firm-specific costs of entering the labor market are realized. Firms that pay the cost post new vacancies, ι_t, such that the total number of vacancies is

$$\tilde{v}_t = \underline{v}_t + \iota_t. \tag{1}$$

Werning (2015), Bilbiie (2019), McKay and Reis (2020), Broer et al. (2020).

Step 2: Search and match. Stochastic matching takes place between unemployed workers and vacancies. The matching technology is Cobb-Douglas, with job-filling rate λ_t^v and job-finding rate λ_t^u

$$\begin{aligned}\lambda_t^v &= \mathcal{M}\theta_t^{-\alpha}, &(2)\\ \lambda_t^u &= \mathcal{M}\theta_t^{1-\alpha}, &(3)\end{aligned}$$

given market tightness

$$\theta_t = \frac{\tilde{v}_t}{\underline{u}_t}. \tag{4}$$

The post-matching labor-market stocks are

$$\begin{aligned}u_t &= (1-\lambda_t^u)\underline{u}_t, &(5)\\ v_t &= (1-\lambda_t^v)\tilde{v}_t. &(6)\end{aligned}$$

Step 3: Production. Production takes place. Dividends and wages are paid.

Step 4: Consumption and saving. All capitalists and workers, both employed and unemployed, make their consumption- and saving decisions.

Step 5: Separations. Vacancies are destroyed with rate $\tilde{\delta}$, which for simplicity we assume to be constant and exogenous. Firms are exposed to an idiosyncratic continuation cost shock and decide whether to continue or exit, which implies an endogenous, time-varying separation rate δ_t. The next period's labor-market states are given by

$$\begin{aligned}1-\underline{u}_{t+1} &= (1-\delta_t)(1-u_t), &(7)\\ \underline{v}_{t+1} &= (1-\tilde{\delta})\tilde{v}_t. &(8)\end{aligned}$$

2.2 Intermediate-good firms, vacancy creation and job separations

There is a continuum of intermediate-good firms producing a homogeneous good sold in a competitive market, owned by the capitalists. The price of the intermediate good is P_t^X and one unit of labor produces A_t units of the intermediate good. The total production of intermediate goods is thus given by

$$X_t = A_t(1 - u_t). \tag{9}$$

To hire labor the firms must post vacancies which are filled with the probability λ_t^v, taken as given by each firm. Denote by V_t^v the value of a vacancy and by V_t^j the value of a match for the firm.

Separations At the end of the period, a firm must pay a continuation cost $\chi_t \sim G$ or else the job match is destroyed. There is no additional heterogeneity, and the expected continuation value is therefore equal across all firms.[7] Hence, there exists a cost cutoff $\chi_{c,t} = \beta \mathbb{E}_t V_{t+1}^j$, such that for all $\chi_t > \chi_{c,t}$, the firm chooses to separate. Accordingly, the Bellman equation for the value of a job prior to the cost draw is given by

$$\begin{aligned} V_t^j &= P_t^X A_t - W_t + \int^{\chi_{c,t}} (\beta \mathbb{E}_t V_{t+1}^j - \chi_t) dG(\chi_t) = \\ &= P_t^X A_t - W_t + (1 - \delta_t)\beta \mathbb{E}_t V_{t+1}^j - \mu_t \end{aligned} \tag{10}$$

where W_t is the real wage, δ_t is the endogenous separation probability given by

$$\delta_t = 1 - G(\chi_{c,t})$$

[7]Following Mortensen and Pissarides (1994), separation decisions are typically modeled as a result of idiosyncratic productivity shocks, such that low-productivity firms decide to exit. Our assumptions have similar material consequences, but avoid ex-post heterogeneity in firm outcomes.

and μ_t is the average cost paid,

$$\mu_t = \int^{\chi_{c,t}} \chi_t dG(\chi_t).$$

We assume that G is a mixture of a point mass and a Pareto distribution with shape parameter ϵ_j such that in steady state, job separations are $\delta_{ss} = \tilde{\delta}$ and the average cost paid is zero, $\mu_{ss} = 0$. Given these assumptions, with details in Appendix A, the following two equations for the endogenous separation probability δ_t and the average cost paid μ_t hold,

$$\delta_t = \tilde{\delta}\left(\frac{\mathbb{E}_t V^j_{t+1}}{V^j_{ss}}\right)^{-\epsilon_j}, \tag{11}$$

$$\mu_t = \tilde{\delta}\frac{\epsilon_j}{\epsilon_j - 1}\beta V^j_{ss}\left(1 - \left(\frac{\mathbb{E}_t V^j_{t+1}}{V^j_{ss}}\right)^{1-\epsilon_j}\right). \tag{12}$$

The idiosyncratic continuation cost implies that the elasticity of job separations to the value of a job is ϵ_j. In the special case where $\epsilon_j = 0$ separations occur exogenously at rate $\tilde{\delta}$.

With search frictions, an additional condition is required to determine how the resulting match surplus is divided. In the baseline model, we follow Hall (2005) and assume that real wages are fixed,[8]

$$W_t = W_{ss}. \tag{13}$$

In our extensions, we consider wage rules that vary with market tightness.

Vacancy creation The Bellman equation for the value of a vacancy is given by

$$V^v_t = -\kappa + \lambda^v_t V^j_t + (1 - \lambda^v_t)(1 - \tilde{\delta})\beta\mathbb{E}_t V^v_{t+1}, \tag{14}$$

[8]The role of wage inertia for unemployment fluctuations in SAM models has been subject ot extensive research. See, e.g., Gertler and Trigari (2009), Rogerson and Shimer (2011), Christiano et al. (2020).

where κ is the flow cost of the vacancy, to be paid every period. Vacancies are not subject to the stochastic continuation cost, and are instead destroyed with exogenous probability $\tilde{\delta}$. There is a constant mass F of prospective firms drawing a stochastic idiosyncratic entry cost c following a distribution H. The prospective firm posts a vacancy if and only if the value of a vacancy is larger than the entry cost. The total number of vacancies created is therefore $\iota_t = F \cdot H(V_t^v)$.

Following Coles and Kelishomi (2018), we assume that the entry-cost distribution has a cumulative distribution function $H(c) = (c/h)^{\epsilon_v}$ on $[0, h]$. With the parameter h sufficiently large so that $h > V_t^v$, the resulting number of vacancies created is $\iota_t = (Fh^{-\epsilon_v}) \cdot (V_t^v)^{\epsilon_v}$. Expressing vacancy creation in relation to steady state gives us

$$\iota_t = \iota_{ss} \left(\frac{V_t^v}{V_{ss}^v} \right)^{\epsilon_v}. \tag{15}$$

The stochastic-cost entry assumption implies that the elasticity of vacancy creation to the value of a vacancy is ϵ_v. In the limit where $\epsilon_v \to \infty$, all entrants pay the same deterministic entry cost and Equation (15) reduces to $V_t^v = V_{ss}^v$. Additionally letting the deterministic entry cost approach zero, we recover the standard free-entry condition, $V_t^v = 0$.

2.3 The final-good sector and the wholesale sector

The representative final-good firm has the production function $Y_t = \left(\int_k Y_{kt}^{\frac{\epsilon_p - 1}{\epsilon_p}} dk \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ where Y_{kt} is the quantity of the input of wholesale firm k's output used in production. The implied demand curve is $Y_{kt} = \left(\frac{P_{kt}}{P_t} \right)^{-\epsilon_p} Y_t$ where $P_t = \left(\int_k P_{kt}^{1-\epsilon_p} dk \right)^{\frac{1}{1-\epsilon_p}}$ is the aggregate price level. There is a continuum of wholesale firms indexed by $k \in [0, 1]$ producing differentiated goods using the production function $Y_{kt} = X_{kt}$ where X_{kt} is the amount of the intermediate good purchased by firm k at the intermediate-good price P_t^X. The wholesale firms face Rotemberg price adjustment costs, with scale factor ξ. Since production is linear, the marginal cost of production is the input price P_t^X. In a symmetric equilibrium, optimal price setting implies a standard Rotemberg Phillips curve

$$1 - \xi + \xi \cdot P_t^X = \xi(\Pi_t - 1)\Pi_t - \beta\xi\mathbb{E}_t\left[(\Pi_{t+1} - \Pi_{ss})\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right] \tag{16}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate, with total output given by

$$Y_t = X_t. \tag{17}$$

2.4 Households

Following Ravn and Sterk (2020), households are of two types: workers and capitalists. Capitalists can buy and sell shares in an equity fund that owns all firms, but does not participate in the labor market. Workers receive wage income W_t if employed and home production income ϑ if unemployed, but cannot buy and sell equity. All households can save in a zero-coupon one-period nominal bond, in zero net supply, which can be purchased at the price $1/(1+i_t)$ where i_t is the nominal interest rate, and face a no-borrowing constraint.

Because of zero net supply and no borrowing, the equilibrium interest rate clears the market only if all households decide not to save, and the borrowing constraint must bind for all but one type of household.[9] The model therefore admits analytical aggregation. Specifically, as in Ravn and Sterk (2020), under the assumption that aggregate shocks are small, the precautionary motive to save against idiosyncratic unemployment risk always gives the employed workers the strongest motive to save, and in equilibrium, the interest rate must only be consistent with their Euler equation,

$$C_{n,t}^{-\sigma} = \beta\mathbb{E}_t\left[\frac{1+i_t}{\Pi_{t+1}}\left\{(1-\mathrm{URISK}_t)C_{n,t+1}^{-\sigma} + \mathrm{URISK}_t C_{u,t+1}^{-\sigma}\right\}\right],$$

where $C_{n,t}$ is the consumption of the employed, $C_{u,t}$ is the consumption of the unemployed,

[9] Formally, any real interest rate low enough such that all three Euler equations are satisfied with weak inequality is consistent with the zero-borrowing limit. The natural interpretation is however to let liquidity approach zero, as in Krusell et al. (2011), then the real interest rate is such that one of the Euler equations holds with equality.

and $\text{URISK}_t = \delta_t(1-\lambda^u_{t+1})$ is the probability that an employed household is unemployed in the next period.

The no-borrowing constraint implies that all households consume their income in equilibrium. Together with the Euler equation for the employed households, this gives us the following asset-market clearing condition,

$$W_t^{-\sigma} = \beta \mathbb{E}_t \left[\frac{1+i_t}{\Pi_{t+1}} \left\{ (1-\text{URISK}_t) W_{t+1}^{-\sigma} + \text{URISK}_t \vartheta^{-\sigma} \right\} \right]. \tag{18}$$

In Appendix A, we formally specify the consumption problems of the capitalists and workers, and derive Equation (18).

2.5 Government

A government sets monetary policy according to a Taylor rule,

$$1+i_t = (1+i_{ss}) \Pi_t^{\phi_\pi - 1} E_t \Pi_{t+1}. \tag{19}$$

The interest rate reacts to both expected and current inflation, such that that real interest rate reacts to current inflation with elasticity $\phi_\pi - 1$. This particular Taylor rule is helpful for explaining the model interactions. Our impulse-response results are close to identical with a standard current-inflation Taylor rule.

2.6 Summary

Equations (1)-(19) describes a closed system of 19 equations in 19 equilibrium variables. In the background, there are equations describing the evolution of profits and consumption of the capitalist, but these variables does not influence the determination of other variables, but follow as residuals. For the impulse-response functions, we solve for a log-linear approximation of the model using Dynare.

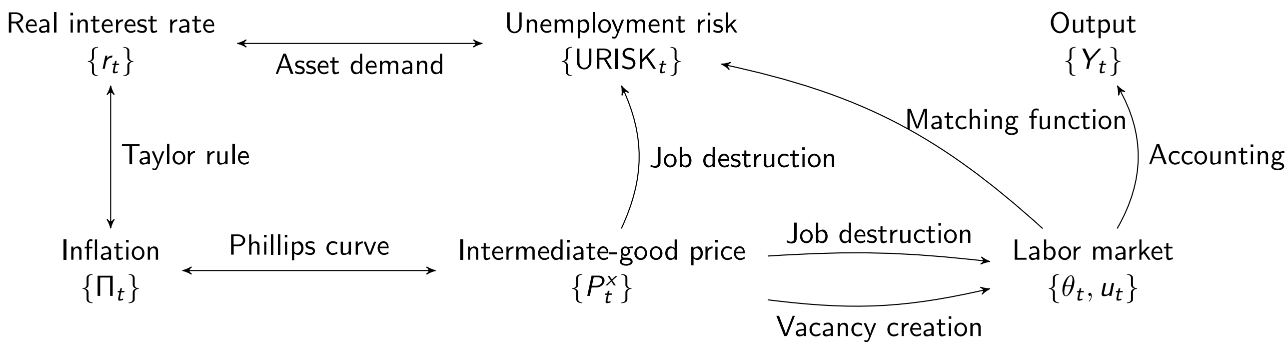

Figure 1: *Graphical representation of the model.*

3 A graphical overview of the model

How does incomplete markets, nominal rigidities and search-and-matching frictions interact to produce a demand-driven feedback mechanism? Figure 1 depicts a graphical representation of the log-linearized approximation of the equilibrium. Each arrow corresponds to an equilibrium relationship. These relationships can be categorized as belong to the HANK block or the SAM block. The HANK block interacts with the SAM block through two variables: the price of intermediate goods $\mathsf{P}^{\mathsf{X}}_{\mathsf{t}}$ and unemployment risk $\mathsf{URSIK}_{\mathsf{t}}$.

Heterogeneous-agent new-Keynesian block. The left-hand side of Figure 1 describes how the new-Keynesian block and the heterogeneous-agent block together generate an aggregate-demand link between unemployment risk and the intermediate-good price. To a first-order approximation, the Phillips curve (16) describes a path of inflation, given a path of marginal cost, or equivalently, a path of intermediate goods prices. The Taylor rule (19) relates inflation to the real interest rate. The path of real interest rates, in turn, must be consistent with the path of unemployment risk through the employed workers Euler equation (18) to make that asset demand equal the fixed zero asset supply. In Appendix A, we show that in a log-linearized equilibrium, these relationships can be collapsed into a single equation relating the path of the intermediate-good price to unemployment risk,

$$\sum_{m \geqslant 0} \beta^m p^x_{t+m} = -\Omega \times \text{urisk}_t, \tag{20}$$

where p^x_{t+m} is the log deviation of P^X_t from steady state and urisk_t is the deviation of URISK_t from steady state. The coefficient Ω captures the strength of the aggregate-demand effect from a change in unemployment risk. It is determined by (i) the strength of the precautionary motive and (ii) the aggregate demand effect from price rigidity and monetary policy,

$$\Omega = \frac{\underbrace{\frac{\left(\frac{W_{ss}}{\vartheta}\right)^{\sigma} - 1}{1 + \text{URISK}_{ss} \times \left(\left(\frac{W_{ss}}{\vartheta}\right)^{\sigma} - 1\right)}}_{\text{Precautionary motive}}}{\underbrace{(\phi_\pi - 1)}_{\text{Monetary policy}} \times \underbrace{(\epsilon_p - 1)\xi^{-1}}_{\text{Price flexibility}}}. \tag{21}$$

If prices are flexible ($\xi \to 0$), if competition is perfect ($\epsilon_p \to \infty$), if monetary policy stabilizes perfectly ($\phi_\pi \to \infty$), or if there is no precautionary motive ($W_{ss} \to \vartheta$ or $\sigma \to 0$), then there is no aggregate-demand feedback from an increase in unemployment risk.

Search and matching block. The right-hand side of Figure 1 describes how the search-and-matching block of the model, given a path of intermediate-good prices, generates labor-market dynamics and unemployment risk.

A fall in the intermediate-good price lowers the profitability of the intermediate-good firms, leading to an increase in job destruction, as seen from Equations (10) and (11). To lower profitability also lowers the value of opening a vacancy, leading to less entry, as seen from Equations (14) and (15). The increase in the job destruction rate directly contributes to an increase in the unemployment risk, and also to an increase in the unemployment rate. The fall in vacancy creation leads similarly leads to an increase in unemployment, and also a fall tightness. The fall in tightness and the higher separation rate results in higher unemployment risk.

General equilibrium. Consider an exogenous fall in labor productivity A_t. Putting the two blocks together, a fall in A_t generates an increase in unemployment risk through the search-and-matching block. From the heterogeneous-agent new-Keynesian block, this increase in unemployment risk generates a fall in the intermediate-good price, which further generates an increase in unemployment risk. Furthermore, dues to the assumption of sluggish entry, the search-and-matching block has considerable persistence, and a temporary therefore generates a persistent increase in unemployment risk, which in turn generates a persistent fall in the intermediate-good price. The endogenous rise in unemployment risk is coupled with a rise in unemployment, which directly determines total output. We turn to study this propagation mechanism quantitatively in the next section.

4 Calibration

For the simulation, we set a model period to a month. Some parameters are set to standard values, some are taken from to match external evidence, and some parameters are internally calibrated such that the zero-inflation steady state of the model matches some key moments in the data.

Table 1 describes the parameters selected without internal calibration. Starting with the households, we set risk aversion σ to 2, in the standard range $[1, 5]$ and the yearly discount factor to 0.96. The home production parameter ϑ determines the consumption drop upon unemployment, given that households cannot borrow nor save in equilibrium. Kolsrud et al. (2018) estimate that in Sweden, the consumption drop is approximately 5 percent, and larger for long-term unemployed. We set ϑ to match the five-percent drop.

For the labor market parameters, we follow Coles and Kelishomi (2018) closely, who estimate a stochastic-cost vacancy-creation model to match key moments of US unemployment dynamics. Note that the flow cost κ is set to zero. With the stochastic entry cost c interpreted as the business investment needed to start a productive opportunity to work, κ should be interpreted as the direct recruiting costs for hiring. Although not easy to estimate, this latter cost is arguably small in relation to the match output. We set the wage level W_{ss}

Parameter	Value	Source
Households		
Risk aversion, σ	2	Standard
Discount factor, β	$0.96^{1/12}$	Standard
Home production, ϑ	$0.95 \times W_{ss}$	Kolsrud et al. (2018)
Labor market		
Wage level, W_{ss}	$0.95 \times \frac{\epsilon_p - 1}{\epsilon_p}$	Hall (2005)
Vacancy creation elasticity, ϵ_v	0.265	Coles and Kelishomi (2018)
Flow vacancy cost, κ	0	Coles and Kelishomi (2018)
Matching elasticity, α	0.6	Coles and Kelishomi (2018)
Price setting and monetary policy		
Substitution elasticity, ϵ_p	6	Standard
Price adjustment cost, ξ	300	Standard
Taylor rule coefficient, ϕ	1.5	Standard

Table 1: *External parameters of the model.*

to be 95 percent of the match output, similar to Hall (2005) and Hagedorn and Manovskii (2008)'s calibrations of a standard free-entry model.

We set the New-Keynesian price-setting and policy parameters to standard values. The slope of the log-linearized Phillips curve is $\frac{\epsilon_p - 1}{\xi}$, which equals 0.017 with our parameters.

Table 2 describes the internally calibrated parameters, which all relates to the labor-market block of the model. The matching productivity $\mathcal{M}$ and the scaling of vacancy cost, F, jointly pin down unemployment duration and market tightness, which we set to the average values for the US reported in Coles and Kelishomi (2018). We set the separation parameter $\tilde{\delta}$, which equals that separation rate in the steady state, to match the average US separation rate, 0.034. Together, these parameters imply a steady-state unemployment rate of 7 percent.

We set the elasticity of the separation rate with respect to job values, ϵ_j, to 0.5, which implies a reasonable response in the separation rate in our impulse-response functions. Going forward, we will calibrate this parameter in a systematic fashion.

Parameter	Value	Steady state target	Value
Matching productivity, $\mathcal{M}$	0.6	Unemp. duration	2.2 months
Scaling of vacancy cost, F	0.7	Tightness	0.5
Separation rate parameter, $\tilde{\delta}$	0.062	E-U transition probability	0.034
Separation elasticity, ϵ_j	0.5	Separation elasticity	0.5

Table 2: *Internally calibrated parameters of the model.*

5 Results

This section studies our economic environment quantitatively, by considering the equilibrium response to a temporary shock to TFP. The exercise is motivated by the economic disruption caused by COVID-19, which has been interpreted as consisting in part of a temporary shock to production capacity (Brinca et al., 2020). More generally, it allows us to highlight in a particularly transparent way the feedback loop that amplifies and propagates exogenous disturbances in our model. To this end, we first study the responses predicted by our baseline model. We then illustrate the complementary nature of the elements in our feedback loop by studying alternative versions of the model where some of them are not active. Finally, we conduct policy experiments to investigate the stabilizing role of transfers to firms.

The shock we consider lowers TFP by one percent for ten months, and then reverts back to steady state:

$$A_t = \begin{cases} 0.99 & \text{for } t = 0, 1, ..., 9, \\ 1.00 & \text{for } t \geqslant 10. \end{cases} \tag{22}$$

5.1 Baseline

Figure 2 displays the impulse-response functions (IRFs) to the temporary TFP shock in our baseline model with sticky prices (black line), and in an alternative version where prices adjust freely ($\xi = 0$) such that real interest rate changes neutralise changes in consumption demand. Figure 3 depicts the differences of these two IRFs, or the "gaps" that capture the

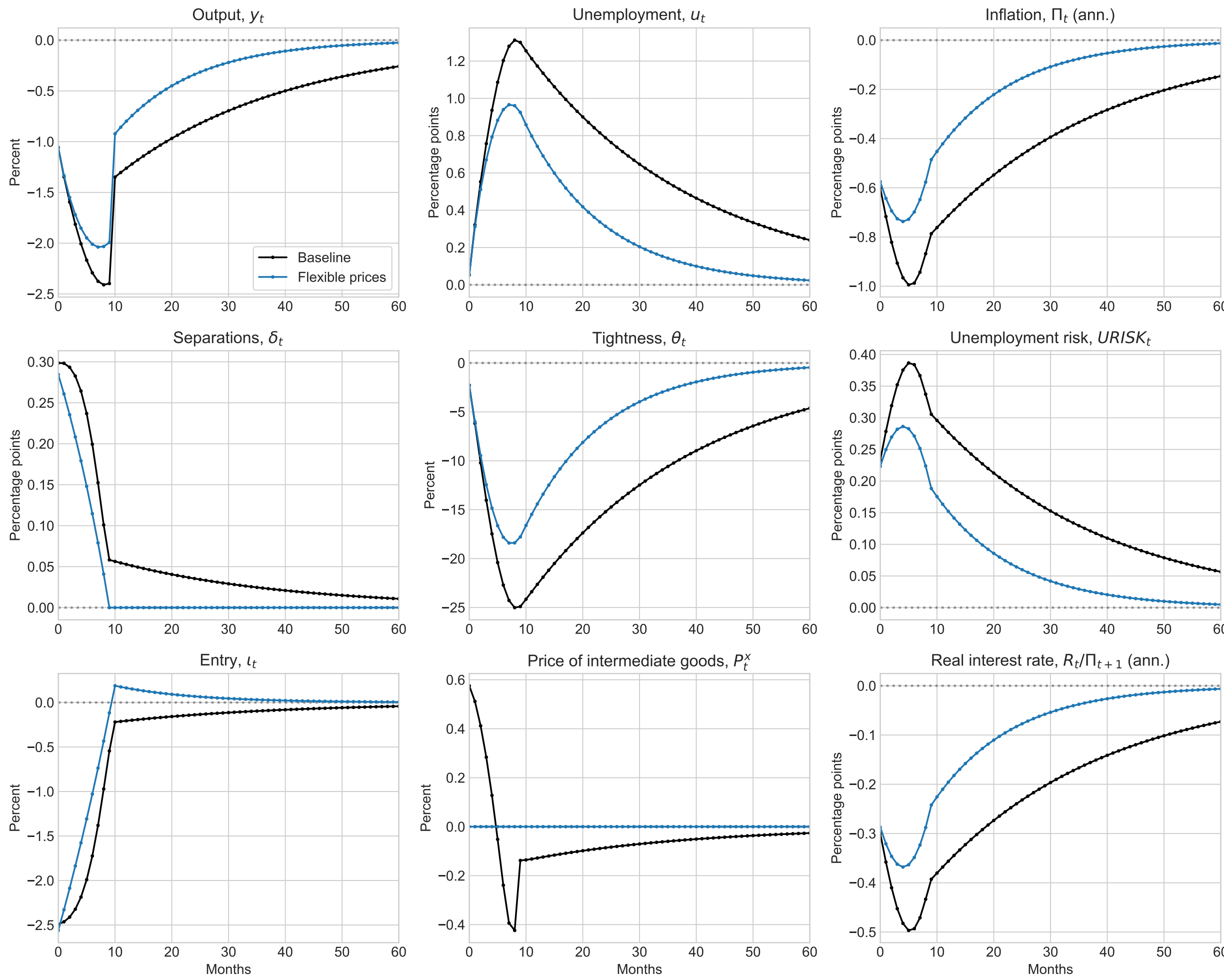

Figure 2: *Impulse-response functions following a one-percent drop in TFP that lasts for 10 months: Baseline model and model with flexible prices*

consequences of nominal rigidities and the conduct of monetary policy in our model.

During the ten months of low productivity, the baseline model in Figure 2 predicts a steep, hump-shaped decline in output and inflation and a steep increase in unemployment, resulting from the sharp increase in separations. Despite a partial recovery when the shocks reverts in period 10, output and inflation remain substantially below their steady state values for more than 4 years. A similar pattern holds for labor market tightness and the job-finding rate, but not for separations, which return to a low level by period 10. As a result of the slow recovery in the job-finding rate, the risk of currently employed workers of being unemployed in the next month also recovers slowly.

Comparing the baseline to the flexible-price model, we see that a similar pattern in most variables arises also with flexible prices, but that all responses are larger and more persistent with sticky prices.

What is the mechanism behind these responses? We start by analyzing the responses of the flexible-price model in Figure 2, which has no aggregate-demand externality. The fall in TFP lowers the net present value of output from firm-worker matches and thus increases separations during the duration of the shock. With flexible prices, however, wholesale sector markups, and therefore the real sales price of the intermediate good are constant. Hence, when TFP returns to its steady state values in period 10, the separation rate also fully recovers. During the period of lower productivity, higher separations increase unemployment (and thus reduce output). Since vacancy creation responds sluggishly, and is reduced by lower productivity, vacancies are depleted by the higher number of unemployed seeking a match. Labor market tightness thus fall, and the job-finding rate that follows the same path, increasing the unmployment risk that workers face. Importantly, tightness and job-finding only recover slowly once productivity returns to normal. The rise in unemployment risk is thus substantially more persistent than the original disturbance, which, through the employed workers' precautionary savings motive, implies a persistent decline in the real interest rate to clear the asset market.

The key difference between this case of flexible prices and our baseline economy with nominal rigidity is the behavior of the real intermediate-good price, which is no longer constant at its steady state value when prices are sticky. Instead, the aggregate-demand link described in Section 3 implies that a rise in unemployment risk must be accompanied by a present-value decline in the intermediate-good price, such that the present value of markups in the wholesale sector increases. The fact that the initial level of the price starts positive can be seen by rewriting Equation (20),

$$p^x_{t+k} = \Omega(\beta \text{urisk}_{t+1} - \text{urisk}_t) \approx \Omega \Delta \text{urisk}_{t+1}.$$

The intermediate-good price is approximately linear in the growth rate of unemployment, which starts positive and then reverts. The persistent decline in the intermediate-good price implies that the value of matches is depressed further, and we therefore get a higher separation rate and lower vacancy creation rate, compared to the flexible-price model. Given a deeper recession in terms of a higher and more persistent separation rate and a lower and more persistent vacancy rate, it follows that the response in all other variables will be amplified and prolonged.

In short, the rise in unemployment risk lowers aggregate demand, which depresses inflation and the value of matches. The depressed match values further deteriorate labor market conditions and increase unemployment risk, generating a demand-driven feedback loop. The demand-driven amplification of the recession can be summarized in terms of the "gap" between the baseline model and the flexible-price model, which we display in Figure 3.

5.2 Which model components drive the propagation mechanism?

Three distinctive features of our model are the assumptions of i) incomplete markets, ii) endogenous separations and iii) sluggish vacancy creation. In Figure 4, we display the IRFs of the gaps when instead assuming i) complete markets (where unemployment risk is fully insured and consumption of the employed and unemployed the same), ii) a constant, exogenous separation rate (equal to the steady state separation rate in the baseline model), iii) free entry into vacancy creation (replacing the entry-equilibrium condition (15) with $V_t^v = 0$ for all t), or iv) free entry and exogenous separations.[10]

With complete markets, the demand-driven recession is much smaller and less persistent. The rise in unemployment risk does not generate a rise in employed workers' precautionary savings and the fall in demand is only due to intertemporal substitution. As a consequence, the real interest rate does not need to fall as much to clear the asset market, and the demand-

[10]Case ii), with a constant separation rate, is equivalent to setting $\epsilon_j = 0$ in our baseline model. In the free-entry cases iii) and iv), we also recalibrate the flow cost of vacancy posting κ; with $\kappa = 0.58$, the free-entry model implies the same steady-state calibration targets.

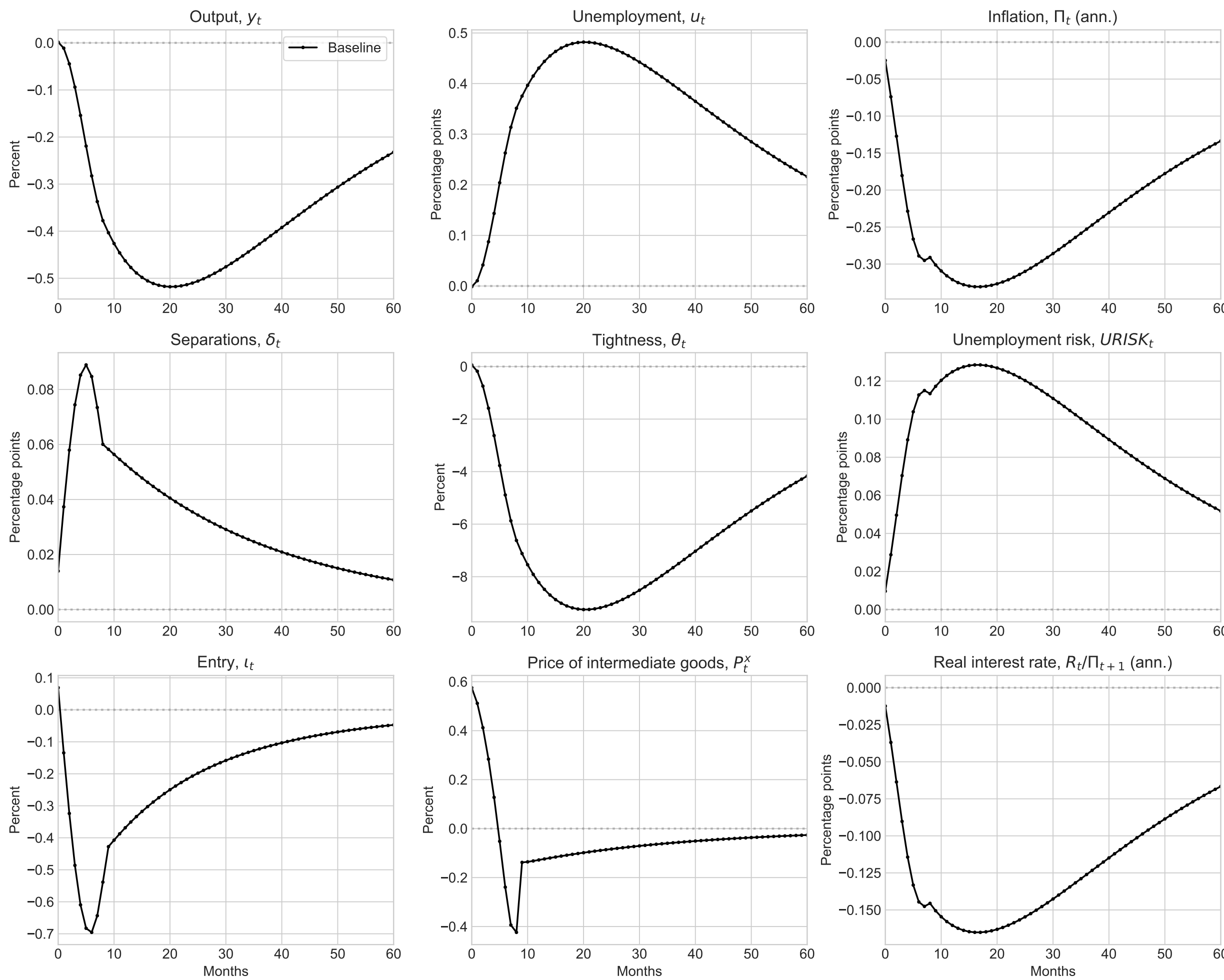

Figure 3: *Impulse-response functions of the baseline model in terms of gaps (deviations from flexible-price model) following a one-percent drop in TFP that lasts for 10 months.*

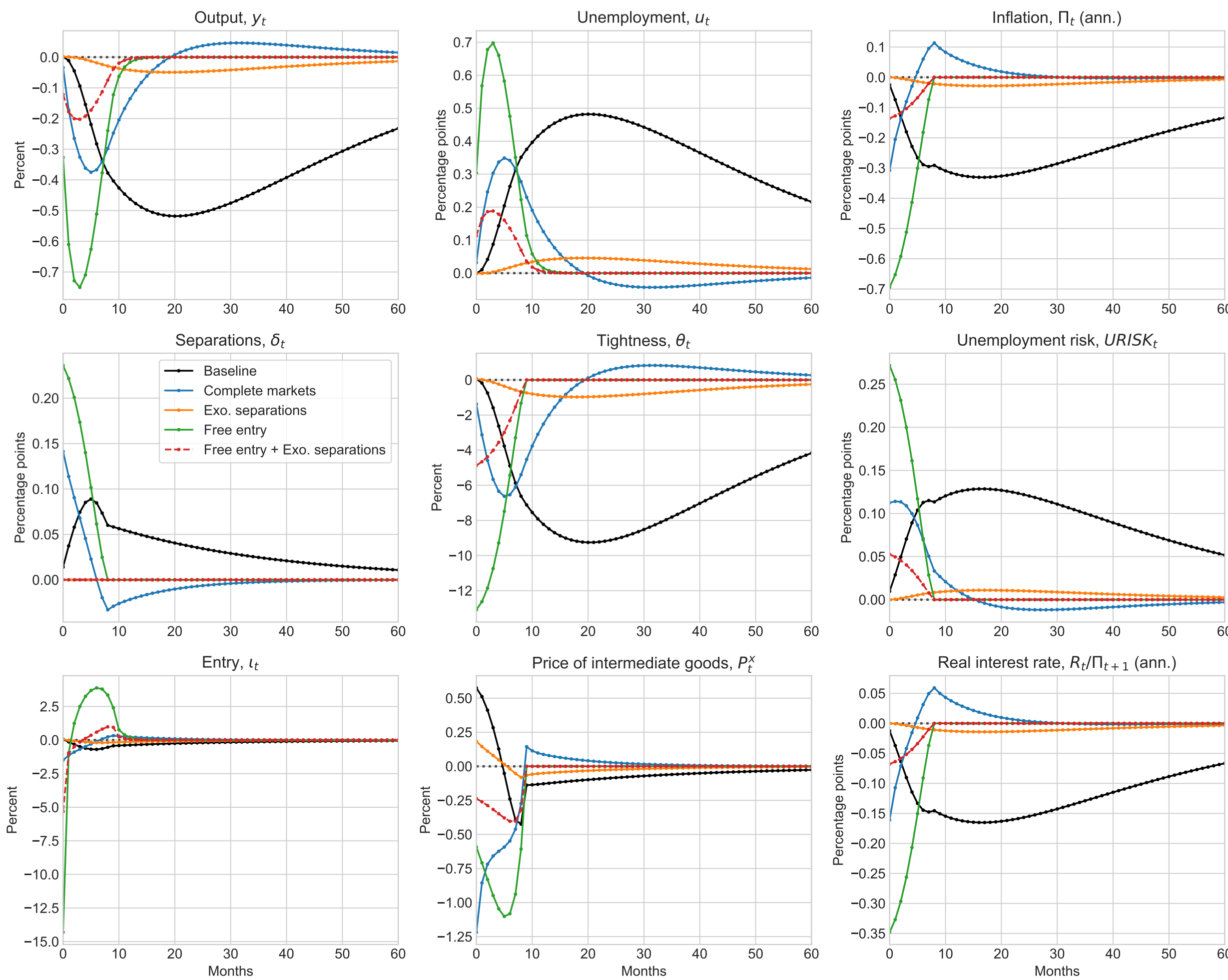

Figure 4: *Impulse-response functions in terms of gaps (deviations from flexible-price model) following a one-percent drop in TFP that lasts for 10 months: baseline model alongside four comparison models.*

driven feedback mechanism is weaker. As a result, the gap in all variables is smaller and less persistent.

With exogenous separations, the responses of all variables are much weaker. The fall in TFP generates a small rise in unemployment through the drop in vacancy creation, but unless separations also rise, unemployment does not grow enough to quickly deplete the current vacancy stock, and the effect on tightness is therefore small. The small response in tightness translates into a small response in job-finding rates and unemployment risk. In turn, the model has a weak response in households' precautionary savings, and there is no strong demand-driven feedback loop.

With free entry, there is no virtually no endogenous persistence in any variable. Since tightness is a jump variable under vacancy creation subject to free entry, the equilibrium system, apart from the evolution of the unemployment rate, is completely forward looking. Intuitively, the initial rise in unemployment generated by the rise in separations is met by a surge in vacancy creation, since, if it did not surge, the larger pool of unemployed households increases the returns to posting a vacancy. Given the lack of persistence in separations and tightness, there is no persistent aggregate-demand feedback loop caused by households' precautionary savings.

Finally, with neither endogenous separations or sluggish entry, the demand-driven amplification is small and there is very little persistence. In its setup, this model is very similar to the model considered by Ravn and Sterk (2020). In its calibration, it is close to Hagedorn and Manovskii (2008), but with added sticky prices.[11]

5.3 Wage cyclicality

Our baseline model assumes, for simplicity, fully rigid real wages. How important is this assumption for the results? To answer this question in a simple way, we replace the constant real wage in Equation (13) with a wage rule that has constant elasticity η with respect to

[11] Hagedorn and Manovskii (2008)'s Nash-bargaining version of the free-entry model similarly has a very high leisure value of unemployed, a high steady state wage level, a moderate level of wage cyclicality and a flow vacancy-posting cost of 0.6.

tightness

$$W_t = W_{ss} \left(\frac{\theta_t}{\theta_{ss}} \right)^{\eta}. \tag{23}$$

A similar wage rule is used in Gornemann et al. (2016). In Figure 5, we compare the IRFs of the gaps in the baseline model to those implied by three different values of the wage elasticity η. With more elastic wages, the gaps in all variables peak at higher levels and have less persistence. The amplified peak response in the first months of the recession comes from the fact that with elastic wages, there is also an intertemporal-smoothing motive in the employed households' saving decision. When real wages are on a downward path, the real interest rate needs to fall further. For this to be consistent with equilibrium there must be an even stronger fall in the intermediate-good prices, which amplifies the response in separations and vacancy creation. The lower persistence comes from the fact that elastic wages mitigate some of the sluggish vacancy creation. If wages fall when unemployment increases, the incentive to post a vacancy becomes even stronger, and more firms will find it worthwhile to pay the vacancy-posting cost.

5.4 Stabilization policy

The analysis in the previous section highlighted the important role of an endogenous rise in separations for generating a deep demand-driven recession. This suggests that policies that dampen this rise in separations might be particularly effective at stabilizing recessions. One way to achieve this are subsidies to firms that keep existing matches alive, such as those that have been used in several countries in response to the recent Covid-19 crisis.

To study the effectiveness of match-saving subsdies as a stabilisation tool, we conduct the following simple policy experiment. We assume that the government taxes the capitalist during the duration of the fall in TFP and transfers the proceeds equally to all firms that currently have a match, maintaining a balanced budget. Specifically, the transfer equals $T_t = \lambda \frac{A_{ss} - A_t}{A_{ss}} Y_{ss}$. The transfer is thus maintained for the duration of the decline in productivity

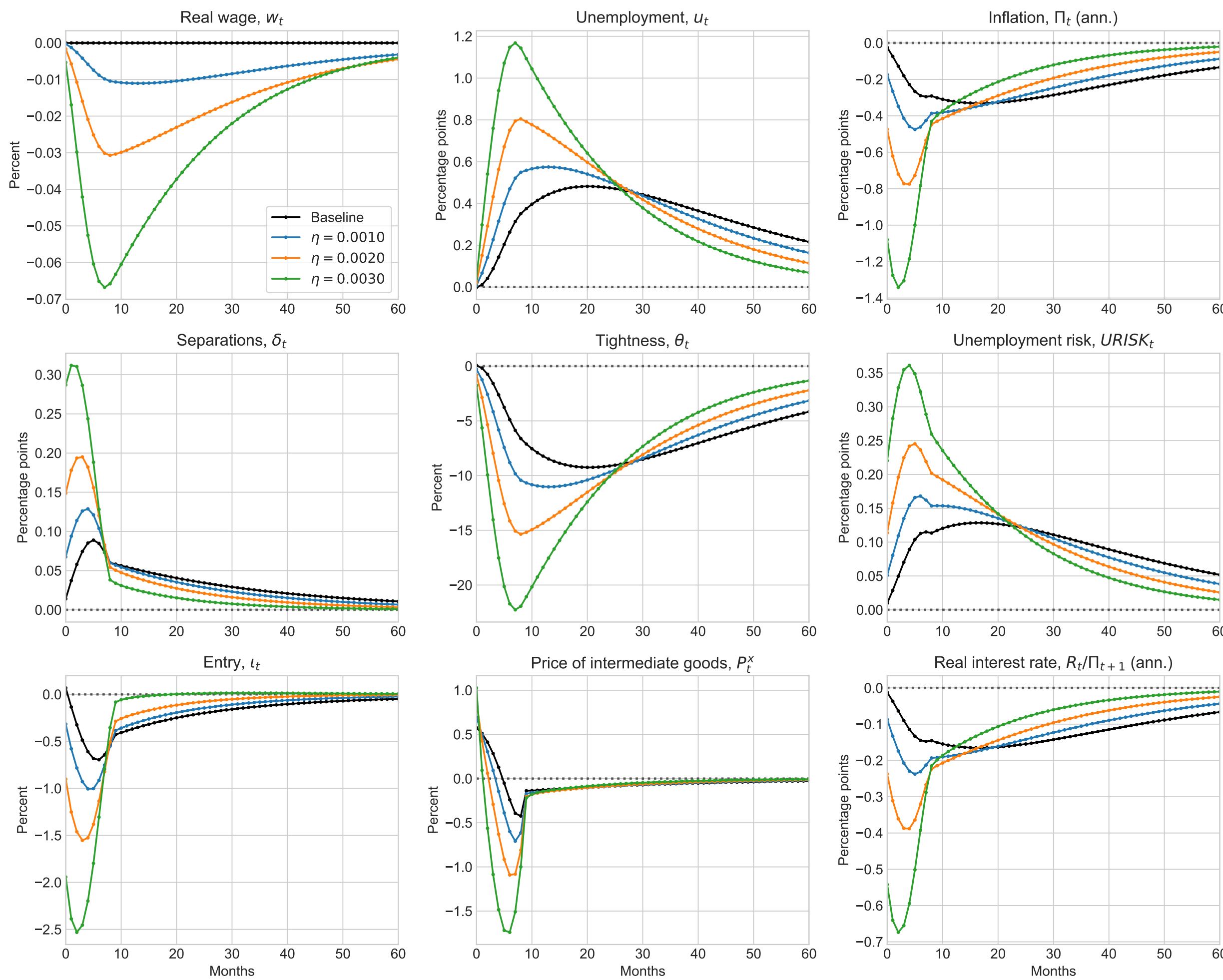

Figure 5: *Impulse-response functions in terms of gaps (deviations from flexible-price model) following a one-percent drop in TFP that lasts for 10 months: baseline model with a fixed real wage level ($\eta = 0$) alongside three comparison models with an elastic real wage.*

and equals a fraction λ of the fall in output that follows directly from the TFP loss. In Figure 6, we show the resulting IRFs, in terms of gaps between the rigid-price and the flexible-price model, with a transfer policy using $\lambda = [0.1, 0.3, 0.5]$ together with the baseline model with no transfer. With the transfer policy in place, the responses of all variables are smaller and the stabilizing effect is increasing in the transfer size. With a large enough transfer, the output gap infact increases in the first period of the shock.

6 Conclusion

Transitory productivity shocks generate long-lived demand deficiencies in an environment with heterogeneous-agent new-Keynesian demand effects and a search-and-matching labor market when (i) job separations respond endogenously to the profitability of matches and (ii) vacancies are created in a sluggish fashion. In such a world, the scope for policy is potentially large. We illustrated this by showing that a match-saving subsidy reduces the severity of the recession. However, a proper cost-benefit analysis of such policies is beyond the scope of our paper at this stage, as our model is missing several ingredients that are relevant for such an evaluation, e.g., persistent firm-specific shocks, a non-degenerate wage distribution, skill loss upon unemployment etc.

Our framework nests a version of the canonical search-and-matching framework (set the job-separation elasticity to zero, $\epsilon_j = 0$, and the vacancy-creation elasticity to infinity, $\epsilon_v = \infty$) so a natural next step is to quantitatively compare the performance of our model against the canonical search-and-matching framework paired with a canonical heterogeneous-agent new-Keynesian block (with positive liquidity and a non-degenerate wealth distribution).

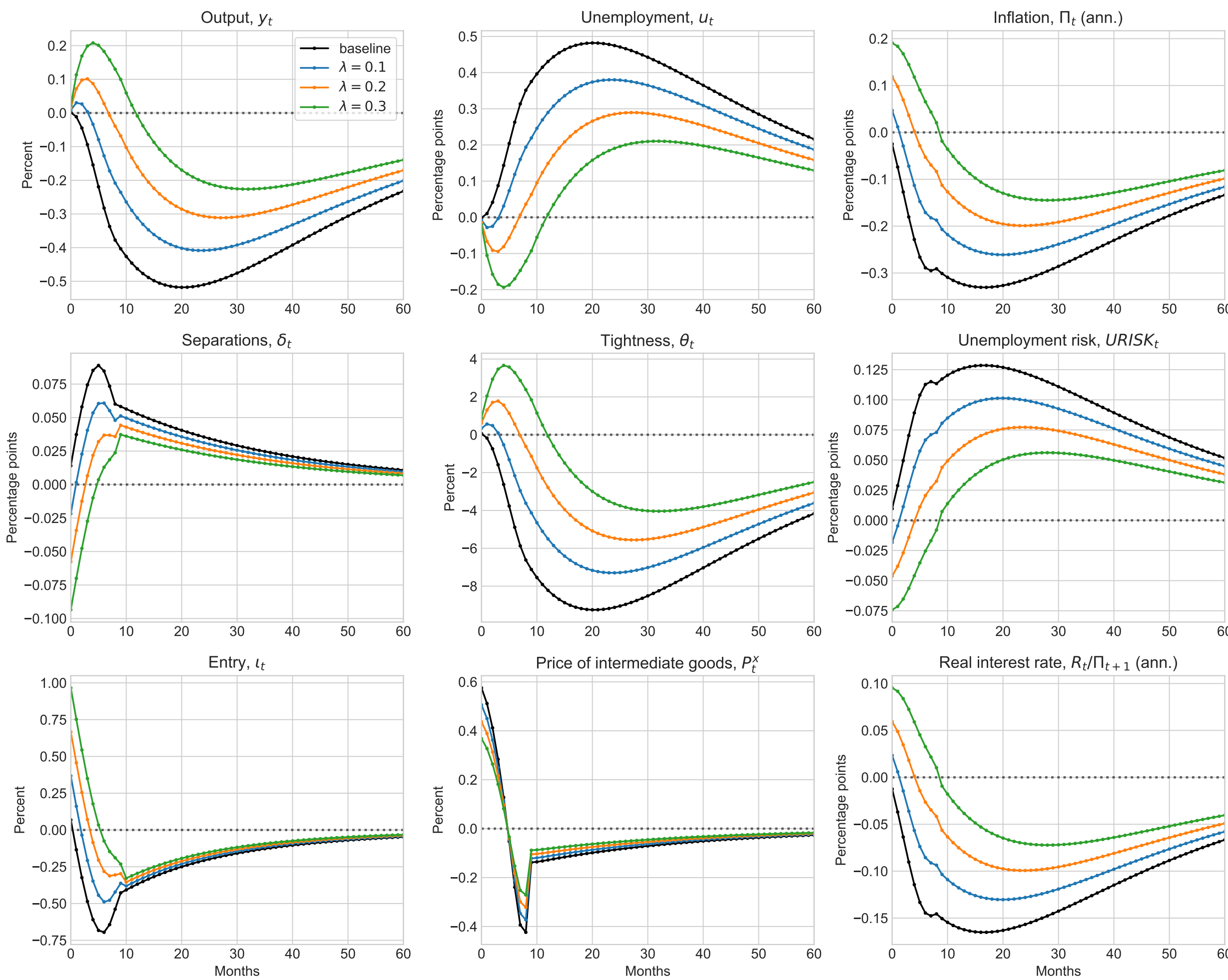

Figure 6: *Impulse-response functions in terms of gaps following a one-percent drop in TFP that lasts for ten months: baseline model and three comparison models with an active firm-subsidy policy.*

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A Model appendix

Deriving the job-separation equations. We assume that G is a mixture of a point mass at $\underline{\chi}$ and a Pareto distribution with location parameter $\overline{\chi}$ (with $\overline{\chi} > \underline{\chi}$) and shape parameter ϵ_j,

$$G(\chi_t) = \begin{cases} 0 & \chi_t < \underline{\chi}, \\ 1-p & \underline{\chi} \leqslant \chi_t < \overline{\chi}, \\ (1-p) + p(1-(\chi_t/\overline{\chi})^{-\epsilon_j}) & \chi_t \geqslant \overline{\chi}, \end{cases} \tag{24}$$

and that the endogenous cutoff is within the support of the Pareto distribution, $\chi_c > \overline{\chi}$. The mass of adjusters is

$$\delta_t = 1 - G(\chi_{c,t}) = p(\chi_{c,t}/\overline{\chi})^{-\epsilon_j} \tag{25}$$

and the average cost paid is

$$\mu_t = (1-p)\underline{\chi} + p\frac{\epsilon_j}{\epsilon_j - 1}\overline{\chi}^{\epsilon_j}(\overline{\chi}^{1-\epsilon_j} - \chi_t^{1-\epsilon_j}). \tag{26}$$

Relating $\delta_t, \mu_t, \chi_{c,t}$ to their steady-state values $\delta_{ss}, \mu_{ss}, \chi_{c,ss}$, we have

$$\delta_t = \delta_{ss}\left(\frac{\chi_{c,t}}{\chi_{c,ss}}\right)^{-\epsilon_j}, \tag{27}$$

$$\begin{aligned} \mu_t &= \mu_{ss} + p\frac{\epsilon_j}{\epsilon_j - 1}\overline{\chi}^{\epsilon_j}(\chi_{c,ss}^{1-\epsilon_j} - \chi_{c,t}^{1-\epsilon_j}) \\ &= \mu_{ss} + \delta_{ss}\frac{\epsilon_j}{\epsilon_j - 1}\chi_{c,ss}\left(1 - \left(\frac{\chi_{c,t}}{\chi_{c,ss}}\right)^{1-\epsilon_j}\right). \end{aligned} \tag{28}$$

By adjusting $\underline{\chi}$ to ensure $\mu_{ss} = 0$ and adjusting p to ensure $\delta_{ss} = \tilde{\delta}$, we arrive at Equations (11)-(12).

Workers' optimization problem The post-decision value function for the employed worker is

$$\mathcal{W}_t^n = \mathbb{E}_t\left[(1-\text{URISK}_t)V_{t+1}^n + \text{URISK}_t V_{t+1}^u\right] \tag{29}$$

where $\text{URISK}_t = \delta_t(1-\lambda_{t+1}^u)$ is the probability of an employed worker becoming unemployed. The Bellman equation for an employed worker is

$$V_t^n = \max_{C_{n,t},B_{n,t+1}} \frac{C_{n,t}^{1-\sigma}}{1-\sigma} - \zeta + \beta \mathcal{W}_t^n \qquad \text{s.t.} \tag{30}$$

$$C_{n,t} + \frac{B_{n,t+1}}{1+i_t} \leqslant W_t + \frac{B_{n,t}}{\Pi_t}, \tag{31}$$

$$B_{n,t+1} \geqslant 0. \tag{32}$$

In the zero-liquidity equilibrium, the sum of all agents' asset holdings is zero. Together with assumption that no agent is allowed to borrow, it follows that all individual agents' asseting holdings must be zero. Hence, $B_{n,t} = B_{n,t+1} = 0$, and all employed workers are symmetrical such that $C_{n,t} = W_t$.

The post-decision value function for the unemployed worker is

$$\mathcal{W}_t^u = \mathbb{E}_t\left[\lambda_{t+1}^u V_{t+1}^n + (1-\lambda_{t+1}^u)V_{t+1}^u\right]. \tag{33}$$

The Bellman equation for an unemployed worker is

$$V_t^u = \max_{C_{u,t},B_{u,t+1}} \frac{C_{u,t}^{1-\sigma}}{1-\sigma} - \zeta + \beta \mathcal{W}_t^u \qquad \text{s.t.} \tag{34}$$

$$C_{u,t} + \frac{B_{u,t+1}}{1+i_t} \leqslant \vartheta + \frac{B_{u,t}}{\Pi_t}, \tag{35}$$

$$B_{u,t+1} \geqslant 0. \tag{36}$$

In the zero-liquidity equilibrium, $B_{u,t} = B_{u,t+1} = 0$, and all unemployed workers are symmetrical such that $C_{u,t} = \vartheta$.

Capitalists' optimization problem The Bellman equation for the capitalists, who do not participate in the labor market, is

$$V_t^c = \max_{C_{c,t}, B_{c,t+1}, C_{t+1}} C_{c,t} + \beta \mathbb{E}_t V_{t+1}^c \qquad \text{s.t.} \tag{37}$$

$$C_{c,t} + \frac{B_{c,t+1}}{1+i_t} + P_t^S S_t \leqslant \vartheta + \frac{B_{c,t}}{\Pi_t} + (P_t^s + D_t) S_{t-1}, \tag{38}$$

$$C_{c,t} \geqslant 0, \tag{39}$$

$$B_{c,t+1} \geqslant 0, \tag{40}$$

$$S_t \geqslant 0, \tag{41}$$

where S_t are equity fund shares. The equity fund owns all firms in the economy, and pays out the firm profits as D_t.

In the zero liquidity equilibrium, $B_{c,t} = B_{c,t+1} = 0$, and with all capitalists symmetrical, $S_t = S_{t+1} = \frac{1}{\text{pop}_c}$. Consumption of the capitalists is given by

$$C_{c,t} = \frac{D_t}{\text{pop}_c} + \vartheta. \tag{42}$$

Since capitalists have linear utility, the discount factor that enter the firm problems is simply β.

Asset market equilibrium Optimality requires that the three Euler equations of the three types of agents are satisfied with weak inequality,

$$W_t^{-\sigma} \geqslant \beta \mathbb{E}_t \left[\frac{1+i_t}{\Pi_{t+1}} \left((1 - \text{URISK}_t) W_{t+1}^{-\sigma} + \text{URISK}_{u,t} \vartheta^{-\sigma} \right) \right], \tag{43}$$

$$\vartheta^{-\sigma} \geqslant \beta \mathbb{E}_t \left[\frac{1+i_t}{\Pi_{t+1}} \left(\lambda_{t+1}^u W_{t+1}^{-\sigma} + (1 - \lambda_{t+1}^u) \vartheta^{-\sigma} \right) \right], \tag{44}$$

$$1 \geqslant \beta \mathbb{E}_t \left[\frac{1+i_t}{\Pi_{t+1}} \right]. \tag{45}$$

Formally, any real interest rate $(1+i_t)/\Pi_{t+1}$ low enough such that all three Euler equations are satisfied with weak inequality is consistent with the zero-liquidity equilibrium. The natural interpretation is however to let liquidity approach zero, as in Krusell et al. (2011), then the real interest rate is such that one of the Euler equations holds with equality.

At a zero-inflation steady state, the three Euler equations amount to

$$1 \geqslant \beta(1+i_{ss}), \tag{46}$$

$$1 \geqslant \beta(1+i_{ss})\left(1+\text{URISK}_{ss}((W_{ss}/\vartheta)^{\sigma}-1)\right), \tag{47}$$

$$1 \geqslant \beta(1+i_{ss})\left(1-\lambda^{u}_{ss}(1-(W_{ss}/\vartheta)^{-\sigma})\right). \tag{48}$$

With the transition rates strictly positive, and the wage of the employed larger than the home production of the unemployed, $W_{ss} > \vartheta$, we get the inequalities $1+\text{URISK}_t((W_{ss}/\vartheta)^{\sigma} - 1) > 1 > 1 - \lambda^{u}_{ss}(1-(W_{ss}/\vartheta)^{-\sigma})$ and the marginal saver is the employed worker. For small enough aggregate shocks, around the zero-inflation steady state, the employed worker remains the marginal saver and Equation (18) is the asset-marking clearing condition.

Deriving the log-linearized HANK block equation In log-linearized terms, the Phillips curve is on the standard form,

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + (\epsilon_p - 1)\xi^{-1}p^{\chi}_t \tag{49}$$

which can be rewritten as

$$\pi_t = \sum_{t' \geqslant t} \beta^{t'-t} p^{x}_{t'}. \tag{50}$$

The specified Taylor is in log-linear terms

$$i_t = (\phi_\pi - 1)\pi_t + \mathbb{E}_t\pi_{t+1} \tag{51}$$

which gives us a Taylor rule in terms of the real interest rate,

$$r_t = (\phi_\pi - 1)\pi_t. \tag{52}$$

A linearization of Equation (18), with $W_t = W_{ss}$, gives

$$r_t = -\frac{\left(\frac{W_{ss}}{\vartheta}\right)^{\sigma} - 1}{1 + \text{URISK}_{ss} \times \left(\left(\frac{W_{ss}}{\vartheta}\right)^{\sigma} - 1\right)} \text{URISK}_t. \tag{53}$$

Equations (52) and (53) give π_t as a multiple of URISK_t. Plugging in to Equation (50) gives Equation (21).