Unemployment Dynamics with Rigid Variable-Pay Contracts*

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Abstract

Do wage-contract rigidity and variable pay matter for the Diamond-Mortensen-Pissarides theory of unemployment dynamics? Hagedorn and Manovskii (2008) suggested using the volatility of real wages and steady-state level of tightness to infer the worker bargaining weight and outside option, and found that with this strategy, the DMP model does explain the observed volatility of unemployment. We assess whether this finding is robust to amending the model with rigid variable-pay contracts, following Broer, Harmenberg, Krusell, and Öberg (2023). For given parameters, neither contract rigidity nor variable pay affect unemployment dynamics, but they do affect wage volatility in response to a productivity shock. With a realistic degree of contract rigidity, and variable-pay contracts calibrated to explain intensive-margin fluctuations in hours worked, the calibrated model cannot explain the observed volatility of unemployment.

1 Introduction

A large literature has investigated the capability of the Diamond-Mortensen-Pissarides (DMP) model of frictional unemployment dynamics to explain the variability of unemployment observed in US data.[1] Ljungqvist and Sargent (2017) identified that the key determinants of the model capability to do so are the model and parameter choices that affect the *fundamental surplus ratio*—the fraction of production that may be allocated to vacancy creation. In the standard DMP model with Nash bargaining, this ratio equals $1-z$, where z is the ratio of workers' outside option and match productivity. Hagedorn and Manovskii (2008), henceforth HM, suggested a calibration strategy for the standard model which targets the observed volatility

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[1]Some important references are Andolfatto (1996), Merz (1995), Shimer (2005), Hall (2005), Hornstein, Krusell, and Violante (2005), Hagedorn and Manovskii (2008) and Hall and Milgrom (2008).

of hourly wages and the steady state level of tightness. Given moderately procyclical wages in the data, this calibration strategy results in a low worker bargaining weight and high value of the outside option of the worker, and thus a small fundamental surplus ratio, implying that the model can explain the observed volatility of unemployment.

In this paper, we investigate whether HM's calibration strategy is robust to extending the standard model with two pervasive features of the US labor market: First, that wage contracts are rigid and, second, that wage contracts allow for some variability in hours worked and pay.[2] Specifically, we integrate the rigid variable-pay contracts in Broer, Harmenberg, Krusell, and Öberg (2023), henceforth BHKO, into a standard DMP labor market model with Nash bargaining and exogenous separations subject to stochastic productivity shocks, and evaluate unemployment volatility following HM's calibration strategy.

BHKO contracts form the solution to a contracting problem in which workers have disutility from working, firms choose hours worked, the firm has limited commitment, and where underlying productivity shocks are not contractable. In this setting, a contract is a wage-hours schedule, from which firms can request more hours worked in exchange for more pay. In this sense, the firm has the "right to manage" and intensive-margin fluctuations in hours worked are "demand determined". In contrast to BHKO, who considered a competitive market for such contracts, we assume that matched workers and firms bargain over such contracts. We also assume that contracts are rigid: incumbent matches face a fixed probability of renegotiation each period, following Calvo (1983). This setup generalizes the standard model of flexible bargaining over fixed-pay-fixed-hours contracts: we recover the standard model under a zero Frisch elasticity when the probability of recontracting equals one.

We first establish that neither contract rigidity, nor variable pay, affect unemployment dynamics, taking parameters as given. In the DMP framework with exogenous separations, unemployment fluctuations result from fluctuations in vacancy creation, which is only affected by wage rigidity among new hires. This result is well known in the literature, see, e.g., Pissarides (2009). Variable pay does not affect unemployment dynamics, because our contracts satisfy an efficiency condition, so that hours worked are set to equalize the marginal rate of transformation with the marginal rate of substitution in response to a productivity shock. By the envelope condition, changes in hours worked to a productivity shock will not affect the profits made by the firms, and thus neither affect vacancy creation.

In contrast, contract rigidity and variable pay do affect wage dynamics, and therefore affect the calibration outcome when targeting wage volatility. Specifically, contract rigidity poses a problem for the model's capability to explain unemployment volatility under HM calibration strategy. With a smaller fraction of contracts renegotiated in response to a productivity shock, the calibration calls for a higher worker bargaining weight to generate enough volatility of average wages, in line with the data. With a higher worker bargaining

[2]For evidence concerning the spread and properties of variable-pay contracts, see Devereux (2001), Swanson (2007), Shin and Solon (2007), Grigsby, Hurst, and Yildirmaz (2021) and Kurmann and McEntarfer (2024).

weight, the worker's outside option must be much worse in order to match steady state tightness, generating a large *fundamental surplus* and less unemployment volatility. With an average renegotiation frequency of 30 weeks, the model-implied standard deviation of unemployment falls by one order of magnitude.

Variable-pay contracts are a potential remedy to this problem, as with more variable pay, the model explains more of the variability of average hourly wages given the worker bargaining weight and contract renegotiation frequency. With variable pay, the calibration thus results in a smaller worker bargaining weight, and, in turn, a higher worker outside option. The variability of within-contract pay is determined by the Frisch elasticity of labor supply. For a unitary Frisch elasticity (an estimate commonly used in the macroeconomic literature) the effect is large, but if calibrated to match the observed fluctuations in average hours worked among the employed, as in our baseline, we find a Frisch elasticity of around 0.15, which is also close to microeconomic estimates of the Frisch elasticity. With this parameter value, variable pay helps, qualitatively, in increasing the volatility of unemployment but quantitatively the model only explains a small fraction of the unemployment variability in the data.

Our findings thus amount to a caution against HM's calibration strategy, which uses moderately procyclical hourly wages as evidence in favor of a high outside option of the worker, and a low fundamental surplus ratio. This finding is sensitive to the assumed frequency of wage renegotiation. Under a realistic degree of wage rigidity, the DMP model calibrated using HM's strategy generates much too little unemployment volatility compared to the data. Adding a realistic degree of variable pay to the negotiated contracts is not sufficient to undo this result.

Other papers have criticized HM's calibration for other reasons. Hornstein, Krusell, and Violante (2005) and Costain and Reiter (2008) show that it leads to implausibly large responses of unemployment to changes in unemployment insurance. Hall and Milgrom (2008) argue that, although HM can replicate unemployment volatility, it does so only by implying a Frisch elasticity of labor supply well above empirical estimates. Our critique is not that HM's calibration implies unreasonable predictions, but rather that the calibration strategy is not robust: once taking into account that wage contracts are infrequently renegotiated, the calibration strategy no longer delivers the high unemployment volatility observed in the data.

In terms of the model components, the closest papers are Gaur, Grigsby, Hazell, and Ndiaye (2024) and Bils, Chang, and Kim (2022). Gaur, Grigsby, Hazell, and Ndiaye (2024) develop a DMP model with dynamic incentive contracts under moral hazard. Firms offer output-contingent pay to elicit effort. Similarly to our paper, they find that due to an envelope condition, intensive-margin fluctuations in pay offsets the fluctuations in productivity, and leave profits and vacancy creation unaffected. Our results show that this finding does not hinge on their particular setup with dynamic incentive contracts, but also applies to our setting without any moral hazard problem. Bils, Chang, and Kim (2022) study a model in which effort is determined through a Nash bargaining problem, allowing effort to vary ex post both because of changes in labor supply and labor demand. In contrast, with our contracts, hours worked is fully determined by labor demand ex post.

2 Model

Our departure point is a standard discrete time search-and-matching model with exogenous separations (Pissarides, 1985, 2000), largely following Hagedorn and Manovskii (2008). Our setup deviates from HM in four dimensions. First, vacancy posting costs are constant. Second, contracts are not rebargained every period, but renegotiated within a match with a constant probability $1-\omega$, following Bils et al. (2022). Third, the firm may vary hours worked n, with workers suffering disutility from working more hours $v(n)$. Fourth, wage contracts are not restricted to a constant hourly wage, but allow for an unrestricted wage-hours schedule $w(n)$, taken as given when firms decide how many hours the worker shall work.

Within a period, timing is the following:

1. Aggregate productivity z is realized
2. Wage contracts are bargained
3. Production and consumption takes place
4. Current matches are separated
5. New vacancies are created
6. New matches are formed

We first describe the basic model environment, then the wage contracting problem, and finally the worker and firm Bellman equations, and other equilibrium conditions, taking the wage contract as given.

2.1 Environment

There is a unit mass of infinitely-lived households that may be either employed or unemployed. All unemployed workers search for jobs. Employed workers who negotiated their contract in period $t-s$ earn labor income $w_{t-s}(n_t)$ in period t while unemployed workers receive a constant flow value of unemployment benefits, denoted by b. There is no savings technology. The household time-0 utility function is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t[c_t - v(n_t)],$$

where $\beta \in (0,1)$ is the intertemporal discount factor, common to workers and firms. If employed, consumption equals the per-period wage payment, $c_t = w_t(n_t)$, n_t denotes the hours of work supplied at time t. If unemployed, $c_t = b$.

An operating firm has a position which is either filled or vacant. A firm with a filled position generates profits equal to $z_t f(n_t) - w_t(n_t)$, given a wage contract $w_t(\cdot)$ and hours worked n_t. The firm unilaterally chooses hours worked ("right to manage") and maximizes profits, taking the wage contract as given. Hence, the problem of maximizing period profits conditional on a wage contract is static:

$$\pi_t = \max_{n_t} z_t f(n_t) - w(n_t).$$

Firms attract workers from the unemployment pool for vacant positions by posting a vacancy at constant cost c. Productivity z_t follows an $AR(1)$ process in logs,

$$\log z_t = \rho \log z_{t-1} + \epsilon_t, \tag{1}$$

where ϵ_t are i.i.d. innovations.

The number of unemployed workers is denoted by u_t, the number of open vacancies by v_t. The total number of successful matches is given by a constant returns to scale matching function $m(u_t, v_t)$, and we define market tightness as $\theta_t = \frac{v_t}{u_t}$. The probability that an unemployed worker finds a job in period t is

$$\lambda_u(\theta_t) = \frac{m(u_t, v_t)}{u_t}. \tag{2}$$

Similarly, an open vacancy matches with an unemployed worker with probability

$$\lambda_v(\theta_t) = \frac{m(u_t, v_t)}{v_t}. \tag{3}$$

Every period, an existing match separates with an exogenous constant probability σ. Employment, e_t, evolves according to the following law of motion,

$$e_{t+1} \quad = \quad (1-\sigma)e_t + m(u_t, v_t), \tag{4}$$

the number of employed workers in the next period equals the number of workers retained from the current period plus the number of new matches formed. Employment and unemployment are related by

$$u_t = 1 - e_t. \tag{5}$$

2.2 Bargaining over rigid variable-pay contracts

Firms and workers Nash bargain over the wage-hours contract $w(\cdot)$ upon forming a match, and also whenever struck by the Calvo fairy. The bargaining problem is

$$\begin{aligned} \max\nolimits_{w_t(\cdot), n_{t+s}} \quad & \left(S_t^W(z_t; w_t)\right)^{\gamma} \left(S_t^F(z_t; w_t)\right)^{1-\gamma}, \\ & n_{t+s} = \arg\max_{n_{t+s}} z_{t+s} f(n_{t+s}) - w_t(n_{t+s}) \ \forall s > 0, \end{aligned}$$

where γ and $1-\gamma$ are the worker and firms' respective bargaining weight. The respective surpluses in the maximization problem are given by

$$\begin{aligned} S_t^F(z_t; w_t) &= \underbrace{\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta(1-\sigma)\omega)^s\left(z_{t+s}f(n_{t+s}) - w_t(n_{t+s})\right)\right]}_{\text{Value during the contract}} + \text{Continuation value}_t^F, \\ S_t^W(z_t; w_t) &= \underbrace{\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta(1-\sigma)\omega)^s\left(w_t(n_{t+s}) - v(n_{t+s}) - b\right)\right]}_{\text{Value during the contract}} + \text{Continuation value}_t^W. \end{aligned}$$

The expressions for the respective continuation values are determined in equilibrium. What is important in this subsection is that the continuation values are independent of the bargained contract.[3]

Characterizing the optimal contract We now proceed to show that the optimal contract is on the form $w_t(n) = v(n) + w_t^{min}$, that is, the wage schedule is the disutility of the worker plus a lump sum transfer.

First, a necessary condition for a contract being optimal is that adding a constant lump sum transfer to the contract should not improve the objective. Note that the constraint of the bargaining problem is not affected by adding a constant to the contract. Writing $w_t^{\epsilon}(n) = w_t(n) + \epsilon$, plugging in w_t^{ϵ} instead of w_t in the objective, differentiating with respect to ϵ, and evaluating at $\epsilon = 0$ yields the following necessary first-order condition,

$$\frac{S_t^W(z_t; w_t)}{S_t^F(z_t; w_t)} = \frac{\gamma}{1-\gamma}. \tag{6}$$

Regardless of the shape of the wage contract, an optimal contract must assign a share γ of the surplus to the worker.

As a result, the solution to the maximization problem is not affected by adding Equation (6) as an additional constraint to the problem. By substituting $S_t^W(z_t; w_t) = \gamma S_t(z_t; w_t)$ and $S_t^F(z_t; w_t) = (1-\gamma)S_t(z_t; w_t)$ (with $S_t(z_t; w_t) = S_t^W(z_t; w_t) + S_t^F(z_t; w_t)$) in the objective, the maximization problem becomes one of maximizing total surplus,

$$\begin{aligned} \max_{w_t(\cdot), n_{t+s}} \quad & S_t(z_t; w_t), \\ & n_{t+s} = \arg\max_n z_{t+s}f(n) - w_t(n) \ \forall s \geq 0, \end{aligned}$$

[3] In equilibrium, the continuation values are given by

$$\begin{aligned} \text{Continuation value}_t^F &= \mathbb{E}_t\left[\sum_{s=1}^{\infty}(\beta(1-\sigma)\omega)^s \frac{1-\omega-\lambda_v(\theta_t)}{\omega} S_{t+s}^F(z_{t+s})\right], \\ \text{Continuation value}_t^W &= \mathbb{E}_t\left[\sum_{s=1}^{\infty}(\beta(1-\sigma)\omega)^s \frac{1-\omega-\lambda_u(\theta_t)}{\omega} S_{t+s}^W(z_{t+s})\right]. \end{aligned}$$

However, in what follows, what matters is not the exact values of the two but that they are exogenous to the match and the contract.

where

$$S_t(z_t;w_t) \quad = \quad \underbrace{\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta(1-\sigma)\omega)^s\left(z_{t+s}f(n_{t+s})-v(n_{t+s})-b\right)\right]}_{\text{Value during the contract}} + \text{Continuation value}_t^F + \text{ Continuation value}_t^W.$$

First, observe that the maximum surplus, in the absence of the constraint, is obtained if and only if the efficiency condition

$$z_{t+s}f'(n_{t+s}) = v'(n_{t+s})$$

holds for all $s \geq 0$. Now, it is also readily seen that if $w_t(n) = v(n) + w_{min}$, then the solution to $\max_{n_{t+s}} z_{t+s}f(n_{t+s}) - w_t(n_{t+s})$ satisfies $z_{t+s}f'(n_{t+s}) = v'(n_{t+s})$.[4] The maximum of the constrained problem thus coincides with the maximum of the unconstrained problem and the solution to the constrained problem is to set $w_t(n) = v(n) + w_{min}$.

Putting the two results together, the optimal contract satisfies

$$w_t(n) = v(n) + w_t^{min}. \tag{7}$$

where w_t^{min} is set so that

$$\frac{S_t^W(z_t;w_t)}{S_t^F(z_t;w_t)} = \frac{\gamma}{1-\gamma}. \tag{8}$$

Thus, all contract vintages satisfy efficiency, so hours worked at time t is shared across matches. We let n_t^* denote hours worked at time t, implicitly defined by

$$z_t f'(n_t^*) = v'(n_t^*). \tag{9}$$

In sum, even though the contracts are not contingent on productivity nor output, they implement the first best allocation by equalizing the marginal variable pay with the marginal disutility of labor. This efficiency property of the contract stems from the fact that both worker and firm utility are linear, and thus transferable across the bargaining parties, and replicates the allocation achieved in a competitive market for contracts, as in BHKO. The difference to BHKO is that here, the "base pay" w^{min} is set to ensure that workers and firms get the surpluses in accordance with their respective bargaining power, whereas in a competitive market, "base pay" is determined by a zero profit condition.

2.3 Bellman equations

When describing the Bellman equations, we take the wage contracts $w_{t-s}(\cdot)$ as given, as well as hours worked at time t for contracts of vintage $t-s$, which we denote by $n_{t|t-s}^*$. Denote the present discounted value of

[4]The converse, that the optimal contract must be on the form $w_t(n) = v(n) + w_{min}$, holds as long as the distribution of z_{t+s} has full support.

a matched firm employing a worker with wage contract w_{t-s} by $J_{t-s}(z_t)$; the value of an unfilled vacancy by $V(z_t)$; the value of an employed worker at wage contract w_{t-s} by $W_{t-s}(z_t)$; and the utility value of an unemployed worker by $U(z_t)$. The Bellman equations for the firm values are

$$J_{t-s}(z_t) = zf(n^*_{t|t-s}) - w_{t-s}(n^*_{t|t-s}) + \beta\,\mathbb{E}\Big[(1-\sigma)\big[\omega J_{t-s}(z_{t+1}) + (1-\omega)J_{t+1}(z_{t+1})\big] + \sigma V(z_{t+1})\Big], \quad (10)$$

$$V(z_t) = -c + \beta\,\mathbb{E}\Big[\lambda_v(\theta_t)\,J_{t+1}(z_{t+1}) + (1-\lambda_v(\theta_t))\,V(z_{t+1})\Big]. \quad (11)$$

which together implies a Bellman equation for firm surplus

$$\begin{aligned} S^F_{t-s}(z_t) \equiv & J_{t-s}(z_t) - V(z_t) = z_t f(n^*_{t|t-s}) - w_{t-s}(n^*_{t|t-s}) + c + \\ & \beta\,(1-\sigma)\omega\mathbb{E}\Big[\big(S^F_{t-s}(z_{t+1}) - S^F_{t+1}(z_{t+1})\big)\Big] + \beta(1-\sigma-\lambda_v(\theta_t))\,\mathbb{E}S^F_{t+1}(z_{t+1}) \end{aligned} \quad (12)$$

The Bellman equations for the worker values are

$$W_{t-s}(z_t) = w_{t-s}(n^*_{t|t-s}) - v(n^*_{t|t-s}) + \beta\,\mathbb{E}\Big[(1-\sigma)\big[\omega W_{t-s}(z_{t+1}) + (1-\omega)W_{t+1}(z_{t+1})\big] + \sigma U(z_{t+1})\Big], \quad (13)$$

$$U(z_t) = b + \beta\,\mathbb{E}\Big[\lambda_u(\theta_t)\,W_{t+1}(z_{t+1}) + (1-\lambda_u(\theta_t))\,U(z_{t+1})\Big], \quad (14)$$

which together implies a Bellman equation for worker surplus

$$\begin{aligned} S^W_{t-s}(z_t) \equiv & W_{t-s}(z_t) - U(z_t) = w_{t-s}(n^*_{t|t-s}) - v(n^*_{t|t-s}) - b + \\ & \beta(1-\sigma)\omega\,\mathbb{E}\Big[S^W_{t-s}(z_{t+1}) - S^W_{t+1}(z_{t+1})\Big] + \beta(1-\sigma-\lambda_u(\theta_t))\,\mathbb{E}\,S^W_{t+1}(z_{t+1}) \end{aligned} \quad (15)$$

2.4 Free entry condition

There is free entry into vacancy posting. This implies that, in equilibrium, the value of an open vacancy for a firm is zero, i.e., $V(z_t) = 0$ for all z_t. By Equation (11), we have

$$\frac{c}{\beta\lambda_v(\theta_t)} = \mathbb{E}J_{t+1}(z_t) = \mathbb{E}S^F_{t+1}(z_{t+1}). \quad (16)$$

3 Equilibrium properties

3.1 Equilibrium labor-market flows

We now show that the unemployment dynamics in our model are, to a first order, identical to the dynamics in a standard DMP model with flexible fixed-pay-fixed-hours contracts, holding the fundamental surplus constant.

In the DMP model, vacancy creation is a result of fluctuations in firm surplus, which, under Nash bargaining, is a constant share of total surplus. Total surplus for matches formed at $t-s$ is defined as

$$S_{t-s}(z_t) \equiv \quad S^F_{t-s}(z_t) + S^W_{t-s}(z_t).$$

Adding up Equations (12) and (15), we have

$$S_{t-s}(z_t) = z_t f(n^*_{t|t-s}) - v(n^*_{t|t-s}) + c - b + \beta(1-\sigma)\omega\,\mathbb{E}\Big[S_{t-s}(z_{t+1}) - S_{t+1}(z_{t+1})\Big]$$
$$+ \beta(1-\sigma)\,\mathbb{E}\,S_{t+1}(z_{t+1}) - \beta\mathbb{E}(\lambda_v(\theta_t)S^F_{t+1}(z_{t+1}) + \lambda_u(\theta_t)S^W_{t+1}(z_{t+1})).$$

Using Equation (16), we get

$$S_{t-s}(z_t) = z_t f(n^*_{t|t-s}) - v(n^*_{t|t-s}) - b + c\theta + \beta(1-\sigma)\omega\,\mathbb{E}\Big[S_{t-s}(z_{t+1}) - S_{t+1}(z_{t+1})\Big]$$
$$+ \beta(1-\sigma-\lambda_u(\theta_t))\,\mathbb{E}\,S_{t+1}(z_{t+1}).$$

The above equation for the surplus for contract vintage $t-s$ was derived under full generality with respect to the shape of the wage contract for vintage $t-s$. We now impose that wage contract optimality, implying efficiency as in Equation (9), and that hours worked is shared across contract vintages:

$$S_{t-s}(z_t) = z_t f(n^*_t) - v(n^*_t) - b + c\theta + \beta(1-\sigma)\omega\,\mathbb{E}\Big[S_{t-s}(z_{t+1}) - S_{t+1}(z_{t+1})\Big]$$
$$+ \beta(1-\sigma-\lambda_u(\theta_t))\,\mathbb{E}\,S_{t+1}(z_{t+1}).$$

Since all contract vintages have the same flow surplus (and share all other parameters), it is immediate that $S_{t-s}(z_t) = S_t(z_t)$ for all $s \geq 0$. We write $S_{t-s}(z_t) = S(z_t)$, arriving at the law of motion for the surplus given by

$$S(z_t) = z_t f(n^*_t) - v(n^*_t) - b + c\theta + \beta(1-\sigma-\lambda_u(\theta_t))\,\mathbb{E}S(z_{t+1}). \tag{17}$$

Finally, invoking the surplus-splitting property of the contract, Equation (8), in Equation (16) yields

$$\frac{c}{\beta\lambda_v(\theta_t)} = \gamma\mathbb{E}S(z_{t+1}), \tag{18}$$

which together with (17) and the constant returns to scale property of the matching function yield

$$S(z_t) = z_t f(n^*_t) - v(n^*_t) - b + \beta(1-\sigma-(1-\gamma)\lambda_u(\theta_t)))\,\mathbb{E}S(z_{t+1}). \tag{19}$$

Equations (18) and (19), together with Equation (9), restated here,

$$z_t f'(n^*_t) = v'(n^*_t), \tag{20}$$

determine the evolution of the surplus $S(z_t)$, labor-market tightness θ_t, and hours worked n^*_t.

Holding n^*_t fixed, Equation (19) is identical to that of a standard DMP model with flexible fixed-pay-fixed-hours contracts, apart from the feature that we explicitly model the disutility of labor. This, together, with the efficiency condition in Equation (20), implies that, to a first order, the dynamics of total surplus and thus labor-market flows are isomorphic to the standard DMP model.

Concretely, linearizing the flow surplus, $z_t f(n_t^*) - v(n_t^*) - b$ around the steady state yields

$$z_t f(n_t^*) - v(n_t^*) - b = z_{ss} f(n_{ss}^*) - v(n_{ss}^*) - b + f(n_{ss}^*) dz_t + \underbrace{[z_{ss} f'(n_{ss}^*) - v'(n_{ss}^*)]}_{=0} dn_t^*$$

where efficiency implies, through the envelope theorem, that the effect of productivity on total surplus is, to a first order, invariant to changes in n_t^*. Rearranging, the elasticity of flow surplus to productivity is given by

$$\frac{z_t f(n_t^*) - v(n_t^*) - b}{z_{ss} f(n_{ss}^*) - v(n_{ss}^*) - b} = \left(1 + \frac{z_{ss} f(n_{ss}^*)}{z_{ss} f(n_{ss}^*) - v(n_{ss}^*) - b}\right) \frac{dz_t}{z_{ss}}.$$

By comparison, in HM, where the disutility of labor is not explicitly modeled, the elasticity of flow surplus to productivity is given by

$$\frac{z_t - b}{z_{ss} - b} = \left(1 + \frac{z_{ss}}{z_{ss} - b}\right) \frac{dz_t}{z_{ss}}.$$

In our setting, $\frac{z_{ss} f(n_{ss}^*) - v(n_{ss}^*) - b}{z_{ss} f(n_{ss}^*)}$ is the *fundamental surplus*. The size of the fundamental surplus scales the impact of a productivity shock on flow surplus, and through the flow surplus the dynamics of labor-market flows, as pointed out by Hagedorn and Manovskii (2008) and Ljungqvist and Sargent (2017). A low surplus ratio implies that steady state profits must be low, which in turn implies that small changes in productivity will have, ceteris paribus, large effects on profits and, in equilibrium, on vacancy creation. The dynamics in our model are, to a first order, identical to the dynamics in a standard DMP model, holding parameters constant (if not explicitly modelling the disutility of labor in the standard model, b must be recalibrated so that the fundamental surplus is identical across the models).

This result echoes that of Gaur, Grigsby, Hazell, and Ndiaye (2024), who similarly showed that the particular shape of an optimal variable-pay contract in the presence of a moral hazard friction does not influence unemployment dynamics. In our setting, the firm chooses hours worked whereas in their setting the worker chooses effort. Although our setting and theirs have different contract environments, they share that the contract is written so as to incentivize the decision maker and in both settings the envelope-theorem logic applies.

3.2 Equilibrium wage dynamics

To solve for equilibrium wage dynamics, we need to determine the level of base compensation, w_t^{min}. We characterize the solution for w_t^{min} to a first order. In Appendix A, we show that the first-order condition of the Nash bargaining problem given by Equation (6) can, to a first order, be written as recursive expression relating the base wage of cohort t, w_t^{min}, and the expected base wage of $t+1$, w_{t+1}^{min}:

$$w_{t-s}^{min} = \frac{1}{\tau} w_{t-s}^{min,*} + \frac{\tau - 1}{\tau} \mathbb{E} w_{t-s+1}^{min}, \tag{21}$$

where $w_t^{min,*}$ is the "target wage" , i.e., the wage that would have been optimal if the parties could bargain every period, satisfying

$$w_t^{min,*} = b + \gamma \left[z_t f(n^*(z_t)) + c\theta_t - (b + v(n^*(z_t))) \right], \tag{22}$$

and where $\tau = \frac{1}{1-\omega\beta(1-\sigma)}$.

Given the solution for each cohort-specific base wage, we can derive a law of motion for aggregate base wage payments in the economy $w_t^{min,agg}$,

$$w_t^{min,agg} = (1-\sigma)\omega w_{t-1}^{min} + [1-(1-\sigma)\omega] w_t^{min}. \tag{23}$$

This expression represents the weighted average of the newly-renegotiated base wage and its lagged value for those matches that survived from the previous period, combined with the base wage of newly-formed matches. Since all cohorts have the same variable component in their wage contracts, aggregate total wage payments is given by

$$w_t^{agg}(z_t) = w_t^{min,agg} + v(n^*(z_t)), \tag{24}$$

where, again, $n^*(z_t)$ is implicitly determined by the efficiency condition (9).

In sum, neither contract rigidity nor variable-pay affect unemployment dynamics holding parameters constant, but as evident from Equations (23) and (24), they do affect equilibrium wage dynamics. These ingredients thus make a difference for the model's capability to explain unemployment dynamics if moments of wage dynamics constitute calibration targets, as in HM. Next, we investigate this quantitatively.

4 Quantitative results

We solve for a first-order approximation around steady state. For all parameter values considered, the equilibrium is unique.

4.1 Calibration

Our calibration proceeds in three steps. First, several parameters are directly taken from HM, based on external empirical estimates and typical values used in the literature. Second, for the new parameters governing disutility of labor and wage rigidity, we provide our own estimates. Third, given these parameters, we estimate the unemployment utility b and the bargaining weight γ following the calibration strategy in HM. In so doing, the model is set at a weekly frequency, following HM's convention of assuming twelve weeks per quarter. We adopt the following functional forms: $f(n_t) = n_t$; $v(n_t) = \kappa \frac{n_t^{1+\psi}}{1+\psi}$; $m(u_t, v_t) = \frac{u_t v_t}{\left(u_t^l + v_t^l\right)^{1/l}}$. The latter ensures that job-finding and vacancy-filling probabilities lie between 0 and 1, while satisfying constant returns to scale. All parameter values are reported in Table 1.

Table 1: Calibrated parameter values.

Parameter	Definition	Value
From HM		
β	Discount factor	$0.99^{1/12}$
l	Matching function parameter	0.3995
c	Cost of posting a vacancy	0.5840
σ	Job separation rate	0.0081
Externally calibrated		
ρ_z	Persistence of productivity process	0.9786
σ_z	Standard deviation of productivity innovations	0.0033
ψ	Inverse of Frisch elasticity	6.3898
ω	Base wage renegotiation probability	0.9555
κ	Scale parameter for disutility from labor when $z^{ss} = 1$	1
Internally calibrated		
b	Value of nonmarket activity	0.4241
γ	Worker's bargaining power	0.5140

Note: This table reports parameter values adopted or calibrated in this paper in the baseline specification.

Parameters taken from HM. The following parameters are determined as in HM. They are either taken as standard values in the literature, inferred from empirical estimates using U.S. labor market data, or calibrated by imposing steady-state conditions consistent with observed features in the data. The discount factor is set to $\beta = 0.99^{1/12}$, which corresponds to a weekly rate implied by a quarterly discount factor of 0.99. The matching function parameter l is calibrated to match the average weekly job-finding rate $\lambda_u = 0.139$ given an estimate of steady state market tightness, $\theta = 0.634$. The separation rate is set to $\sigma = 0.0081$. Together, these values imply a steady-state unemployment rate of $u = \frac{\sigma}{\sigma+\lambda_u} \approx 0.0551$.[5]

Productivity process. Labor productivity, z_t, is measured in the data using quarterly, seasonally adjusted real output per total hours worked in the U.S. nonfarm business sector, as reported by the Bureau of Labor Statistics (BLS) (FRED series: OPHNFB). For the sample period 1951:Q1 to 2004:Q4, we compute an autocorrelation of 0.692 and an unconditional standard deviation of 0.0106 for the HP-filtered log productivity series (using a smoothing parameter of 1600).

In the model, productivity is assumed to follow an AR(1) process in logs, as described in Equation 1, with

[5] When implementing the method in HM using the same target moments for calibrating l, we find a value slightly different than that reported in HM. Our value is 0.3995, whereas HM use 0.407.

mean $\log z = 1$, autocorrelation parameter ρ_z, and innovations follow a standard normal distribution with standard deviation σ_z. We calibrate ρ_z and σ_z of our weekly process for z such that the quarterly averages of the model-generated data match the quarterly empirical moments. For that, the model-simulated series is aggregated to quarterly frequency, logged, and HP-filtered. This calibration implies a weekly autocorrelation $\rho_z = 0.9786$ and a standard deviation for the shocks of $\sigma_z = 0.00333$.

Labor disutility and contract rigidity. We calibrate the Frisch elasticity of labor supply $\frac{1}{\psi}$ by exploiting the optimality condition for hours worked implied by the model. Given an optimal contract, the marginal rate of substitution equals the marginal rate of transformation $v'(n_t) = z_t f'(n_t)$. Under our parametric assumptions, in logs this condition is

$$\log n_t = \frac{1}{\psi} \log z_t - \frac{1}{\psi} \log \kappa.$$

We thus recover the Frisch elasticity, $1/\psi$ from the slope coefficient from a regression of log hours worked on log labor productivity.

We estimate the Frisch elasticity using the BLS's average weekly hours (FRED: PRS85006023). The resulting regression coefficient is 0.1565, which implies $\psi = 6.39$, corresponding to a Frish elasticity of 0.16. This value is in line with microeconomic estimates in the empirical literature, see, e.g., Martinez, Saez, and Siegenthaler (2021) and Sigurdsson (2020).

The scale parameter κ in the disutility of labor is calibrated to ensure that steady-state output is identical across specifications. Specifically, κ is set such that in steady state, total hours worked sum to one, $n_{ss} = 1$. With our functional forms, the efficiency condition in Equation (9) is $n = \left(\frac{z}{\kappa}\right)^{1/\psi}$, and we thus get $\kappa = z_{ss}$.

We set the contract duration parameter $\omega = 0.9555$, which corresponds to an average contract duration of approximately 22.5 weeks. While a contract duration of three quarters (36 weeks) would better align with typical empirical estimates, the calibration of ψ imposes a practical upper bound on the contract stickiness parameter ω: $\omega = 0.9555$ is the highest feasible value for ω before the worker's outside option b becomes negative under $\psi = 1$, which is the highest value of that parameter considered in the results section.

Unemployment utility and bargaining weight. The two remaining parameters—the worker's value of nonmarket activity b and the worker's bargaining power γ—are jointly calibrated following the strategy proposed by HM. Specifically, b is set to match the average level of market tightness, $\theta = 0.634$ as reported in HM, and the bargaining weight γ is calibrated to match the empirically observed elasticity of wages with respect to productivity.

To estimate this elasticity, we regress the HP-filtered log of real hourly wages on the HP-filtered log of real labor productivity. We find a wage elasticity of 0.4064 for the period 1951:Q1 to 2004:Q4. Hourly

Table 2: Simulation and calibration results across parameter configurations.

	$\omega = 0.956$, $\psi = 6.390$ **(Baseline)**			$\omega = 0.956$, $\psi \to \infty$			$\omega = 0.956$, $\psi = 1$		
Measure	No-var pay	Our model	Diff. (%)	No-var pay	Our model	Diff. (%)	No-var pay	Our model	Diff. (%)
b	0.3685	0.4241	–	0.5038	0.5038	–	0.0038	0.2276	–
γ	0.5452	0.5140	–	0.5452	0.5452	–	0.5452	0.3868	–
Fund. surplus ratio	0.4962	0.4406	-11.21	0.4962	0.4962	0.00	0.4962	0.2724	-45.10
std(u)	0.0097	0.0109	12.37	0.0097	0.0097	0.00	0.0097	0.0174	79.38
	$\omega = 0$, $\psi = 6.390$			$\omega = 0$, $\psi \to \infty$ **(HM Replication)**			$\omega = 0$, $\psi = 1$		
Measure	No-var pay	Our model	Diff. (%)	No-var pay	Our model	Diff. (%)	No-var pay	Our model	Diff. (%)
b	0.8172	0.8175	–	0.9525	0.9525	–	0.4525	0.4548	–
γ	0.0567	0.0559	–	0.0567	0.0567	–	0.0567	0.0516	–
Fund. surplus ratio	0.0475	0.0472	-0.63	0.0475	0.0475	0.00	0.0475	0.0452	-4.84
std(u)	0.0848	0.0853	0.59	0.0848	0.0848	0.00	0.0848	0.0883	4.13

wages are constructed as the product of the labor share (FRED: PRS85006173) and real labor productivity.[6] To ensure consistency, we compute the model-implied elasticity of wages using the same procedure as in the data: after generating the quarterly wage series from the model (as done for productivity), we regress HP-filtered log real wages on HP-filtered log productivity to recover the implied $\varepsilon_{w,p}$, and adjust γ until it matches the empirical target.

4.2 Unemployment dynamics

In Table 2, we display the resulting unemployment volatility for our baseline calibration together with the calibrated parameter values for γ and b and the implied fundamental surplus ratio. As seen in the Table, the resulting standard deviation of unemployment in our model is 0.01, far below the empirical standard deviation around 0.08. Evidently, calibrating our model in the same manner as HM does not overcome "the unemployment volatility puzzle".

To understand what model and calibration choices explain the difference between the unemployment volatility in our model and in HM, Table 2 also displays the analogous numbers for the corresponding model without variable pay, and for several other comparison models where we vary the contract rigidity parameter ω and the labor supply disutility parameter ψ. All models are recalibrated to match steady state tightness and the elasticity of wages with respect to productivity. In the "no variable pay" model, hours

[6] Alternatively, one could estimate the wage elasticity using the BLS's direct measure of real hourly compensation (FRED: COMPRNFB), which yields an estimated elasticity of 0.4329.

worked is constant within a match and the wage payment is fully determined by Nash bargaining either at match formation or at the time of renegotiation. The version of the "no variable pay" model with fully flexible wages, $\omega = 0$, and constant hours worked, $\psi \to \infty$ (Frisch elasticity $\to 0$), is a near replication of HM, in which there is no unemployment volatility puzzle. The only difference to HM is the slightly different calibration targets (reflecting that we measure productivity as output per hour, instead of output per worker) and that we abstract from fluctuations in vacancy posting costs.

Comparing the top three panels with the bottom three panels in Table 2, we see that by making wage contracts rigid, the calibration results in a higher fundamental surplus ratio, and lower unemployment volatility. Ceteris paribus, more rigid contracts make total wages less volatile, and targeting the same wage volatility thus calls for higher bargaining weight γ. With a higher bargaining weight, the unemployment utility flow b is smaller to match the same level of steady state tightness.

Comparing the two columns within each parameter configuration, we see that allowing for variable hours and pay in the contracts results in a lower fundamental surplus ratio, and higher unemployment volatility. Ceteris paribus, variable pay amplifies fluctuations in total wage payments, and the calibration thus calls for a lower bargaining weight γ and a higher unemployment utility flow b.

However, by again comparing across rows, we see that variable pay only makes a quantitatively meaningful difference if wage contracts are sufficiently rigid. With flexible rebargaining, the marginal effect of variable pay on wage volatility is small, and does therefore not affect the calibration outcome significantly. Moreover, by comparing columns within the top panels, we see that given rigid wage contracts, variable pay only makes a quantitatively meaningful difference if the Frisch elasticity $1/\psi$ is high enough. In the limit $\psi \to \infty$, hours worked do not respond to changes in the marginal wage, and by implication, the within-contract wage payment is held constant. For a unitary Frisch elasticity (a commonly used value in the macroeconomic literature), the effect is very large, almost doubling the standard deviation of unemployment.

In our baseline, $\psi = 6.39$, implying a Frisch elasticity of 0.156. This is substantially smaller than the common macroeconomic estimate of unity, reflecting that fluctuations in hours worked at the intensive margin is significantly smaller than fluctuations in total hours worked (the latter is the natural target for a model with no distinction between the intensive and the extensive margin). With this value, the standard deviation of unemployment increases by 12 percent when allowing for variable pay, which is meaningful, but far from sufficient to restore the low unemployment volatility implied by rigid wage contracts under this calibration strategy.

5 Conclusion

Our findings amount to a caution against interpreting HM's finding that moderately procyclical hourly wages as evidence in favor of a high outside option of the worker, and a low fundamental surplus ratio. This finding is sensitive to the assumed frequency of wage renegotiation, and adding a realistic degree of variable pay to the negotiated contracts is not sufficient to undo this sensitivity.

We reached this conclusion using a particular model for variable pay. In the data, workers' pay may not only reflect compensation for hours worked but also effort. Cyclical fluctuations in effort would also contaminate our estimates of the exogenous fluctuations in labor productivity. Although there is a paucity of empirical measures of aggregate effort, it would be interesting to revisit the exercise in this paper where the driving process for labor productivity and the wage contract parameters are set to also match cyclical fluctuations in effort.

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A Solving for the base wage $w_t^{\min}$

This appendix derives an expression for the base wage component, $w_t^{\min}$, which is negotiated at the start of a match in period t as part of a contract that splits the surplus according to Nash bargaining.

Consider the surplus of a firm belonging to cohort t, Equation (12), given by:

$$\begin{aligned} S_t^F(z_t) = &\, z_t f(n^*_{t|t}) - w_t(n^*_{t|t}) + c + \\ &\beta\,(1-\sigma)\omega\mathbb{E}\Big[\big(S_t^F(z_{t+1}) - S_{t+1}^F(z_{t+1})\big)\Big] + \beta(1-\sigma-\lambda_v(\theta_t))\,\mathbb{E}S_{t+1}^F(z_{t+1}) \end{aligned}$$

Taking a first-order Taylor approximation of $S_t^F(z_{t+1})$, the firm's surplus in t+1 for firms of cohort t, around w_{t+1}^{min}, we obtain:

$$S_t^F(z_{t+1}) - S_{t+1}^F(z_{t+1}) \approx \left.\frac{\partial S_t^F(z_{t+1})}{\partial w_t^{\min}}\right|_{w_t^{\min}=w_{t+1}^{\min}} (w_t^{\min} - w_{t+1}^{\min}). \tag{25}$$

Define $\mu_t(z_{t+1})$ as the (minus) marginal effect of the base wage negotiated in period t on the continuation value of a firm from cohort t in state z_{t+1}. Formally:

$$\mu_t(z_{t+1}) = -\left.\frac{\partial S_t^F(z_{t+1})}{\partial w_t^{\min}}\right|_{w_t^{\min}=w_{t+1}^{\min}} \tag{26}$$

Taking the derivative of $S_t^F(z_{t+1})$ with respect to $w_t^{\min}$, evaluated at $w_t^{\min} = w_{t+1}^{\min}$, and applying the expression recursively, yields:

$$\begin{aligned} \mu_t(z_{t+1}) &= -\left[-1 - \beta(1-\sigma)\omega\,\mathbb{E}\left(\left.\frac{\partial S_t^F(z_{t+2})}{\partial w_t^{\min}}\right|_{w_t^{\min}=w_{t+1}^{\min}}\right)\right] \\ &= 1 + \beta(1-\sigma)\omega\,\mathbb{E}\mu_t(z_{t+2}). \end{aligned} \tag{27}$$

Substituting this into the firm surplus yields:

$$\begin{aligned} S_t^F(z_t) = &\, z_t f(n^*_{t|t}) - w_t(n^*_{t|t}) + c + \\ &\beta(1-\sigma)\omega\,\mathbb{E}\left[\mu_t(z_{t+1})(w_t^{\min} - w_{t+1}^{\min})\right] + \beta(1-\sigma-\lambda_v(\theta_t))\,\mathbb{E}S_{t+1}^F(z_{t+1}). \end{aligned} \tag{28}$$

Now consider the surplus in Equation (15) for a worker belonging to cohort t:

$$\begin{aligned} S_t^W(z_t) = &\, w_t(n^*_{t|t}) - v(n^*_{t|t}) - b + \\ &\beta(1-\sigma)\omega\,\mathbb{E}\Big[S_t^W(z_{t+1}) - S_{t+1}^W(z_{t+1})\Big] + \beta(1-\sigma-\lambda_u(\theta_t))\,\mathbb{E}\,S_{t+1}^W(z_{t+1}) \end{aligned}$$

Taking a first-order Taylor approximation of $S_t^W(z_{t+1})$, the worker's surplus in t+1 for firms of cohort t, around w_{t+1}^{min}, we obtain:

$$S_t^W(z_{t+1}) - S_{t+1}^W(z_{t+1}) \approx \left.\frac{\partial S_t^W(z_{t+1})}{\partial w_t^{min}}\right|_{w_t^{min}=w_{t+1}^{min}} (w_t^{min} - w_{t+1}^{min}) \tag{29}$$

Define $\epsilon_t(z_{t+1})$ as the marginal effect of the base wage negotiated in period t on the continuation value of a worker from cohort t in state z_{t+1}. We get:

$$\epsilon_t(z_{t+1}) = \left.\frac{\partial S_t^W(z_{t+1})}{\partial w_t^{\min}}\right|_{w_t^{\min}=w_{t+1}^{\min}}. \tag{30}$$

To derive its recursive form, we differentiate $S_t^W(z_{t+1})$ with respect to $w_t^{\min}$, evaluated at $w_t^{\min} = w_{t+1}^{\min}$:

$$\begin{aligned}\epsilon_t(z_{t+1}) &= 1 + \beta(1-\sigma)\omega\,\mathbb{E}\left[\left.\frac{\partial S_t^W(z_{t+2})}{\partial w_t^{\min}}\right|_{w_t^{min}=w_{t+1}^{min}}\right] \\ &= 1 + \beta(1-\sigma)\omega\,\mathbb{E}\epsilon_t(z_{t+2}).\end{aligned} \tag{31}$$

Thus, the worker surplus becomes:

$$\begin{aligned}S_t^W(z_t) = {} & w_t(n_{t|t}^*) - v(n_{t|t}^*) - b + \\ & \beta(1-\sigma)\omega\,\mathbb{E}\left[\epsilon_t(z_{t+1})(w_t^{\min} - w_{t+1}^{\min})\right] + \beta(1-\sigma-\lambda_u(\theta_t))\,\mathbb{E}\left[S_{t+1}^W(z_{t+1})\right].\end{aligned} \tag{32}$$

We now characterize the determination of the base wage $w_t^{\min}$ through Nash bargaining. The base wage $w_t^{\min}$ is set to divide the total match surplus between the worker and the firm according to the worker's bargaining weight γ. Denoting the total surplus from the match as the sum of the individual surpluses $S_t^F(z_t) + S_t^W(z_t)$, the Nash solution implies the standard surplus-sharing condition of Equation (8):

$$(1-\gamma)S_t^W(z_t) = \gamma S_t^F(z_t). \tag{33}$$

To solve for $w_t^{\min}$, we combine the expressions above and use the following auxiliary relationships:

1. $\epsilon_t(z_{t+1}) = \mu_t(z_{t+1})$, i.e., the marginal increase in worker's surplus equals the negative of the marginal decrease in firm's surplus due to an increase in base wage in period t,
2. Wage schedule: $w_t(n) = v(n) + w_t^{min}$,
3. From the definition of the market tightness, $\theta_t = \frac{v_t}{u_t}$., along with Equations (2) and (3), we have that $\lambda_u(\theta_t) = \theta_t\lambda_v(\theta_t)$,
4. Free-entry condition in Equation (16): $\frac{c}{\beta\lambda_v(\theta_t)} = \mathbb{E}S_{t+1}^F(z_{t+1})$,
5. The surplus-splitting property of firms and workers of cohort $t+1$: $\mathbb{E}S_{t+1}^W(z_{t+1}) = \frac{\gamma}{1-\gamma}\mathbb{E}S_{t+1}^F(z_{t+1})$.

Substituting Equations (28) and (32) into the Nash bargaining condition (33), and using the equations above, the base wage $w_t^{\min}$ satisfies:

$$w_t^{\min} = \frac{\gamma\left[z_t f(n_{t|t}^*) + c\theta_t\right] + (1-\gamma)\left[v(n_{t|t}^*) + b\right] - w_t^v(n_{t|t}^*) + \beta(1-\sigma)\omega\,\mathbb{E}\left[\epsilon_t(z_{t+1})w_{t+1}^{\min}\right]}{1 + \beta(1-\sigma)\omega\,\mathbb{E}\left[\epsilon_t(z_{t+1})\right]}, \tag{34}$$

where $w_t^v(n_t)$ corresponds to the variable pay component in period t, and it is equal to the disutility of the worker, $v(n_t)$.

In a fully flexible wage environment ($\omega = 0$), the base wage reduces to the standard Nash bargaining solution:

$$w_t^{\min,*} = b + \gamma\left[z_t f(n^*(z_t)) + c\theta_t - (b + v(n^*(z_t)))\right] \tag{35}$$

where $w_t^{min,*}$ is the "target wage" , i.e., the wage that would have been optimal if bargaining occurred every period, and where hours worked in period t, n_t^*, is implicitly defined by Equation (9).

With $\epsilon_t(z_t) = \frac{\partial S_t^W(z_t)}{\partial w_t^{\min}} = 1 + \beta(1-\sigma)\omega\,\mathbb{E}\big[\epsilon_t(z_{t+1})\big]$, we can rewrite the Nash-bargained base wage in Equation (34) as a convex combination of the flexible-wage benchmark $w_t^{\min,*}$ and the expected future base wage:

$$w_t^{\min} = \frac{1}{\epsilon_t(z_t)} w_t^{\min,*} + \left(\frac{\epsilon_t(z_t) - 1}{\epsilon_t(z_t)}\right)\mathbb{E}\big[w_{t+1}^{\min}\big]. \tag{36}$$

The weight depends on $\epsilon_t(z_t)$, which admits the following closed-form expression:

$$\begin{aligned}
\epsilon_t(z_t) &= 1 + \beta(1-\sigma)\omega\,\mathbb{E}\big[\epsilon_t(z_{t+1})\big] \\
&= \sum_{j=0}^{\infty}\big[\beta(1-\sigma)\omega\big]^j \\
&= \frac{1}{1-\beta(1-\sigma)\omega},
\end{aligned} \tag{37}$$

therefore, a higher $\epsilon_t(z_t)$ places more weight on the expected future wage, reflecting greater inertia in base wage adjustment due to longer base wage contracts.