Macroeconomic Dynamics with Rigid Wage Contracts

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Abstract

We adapt the wage contracting structure in Chari (1983) to a dynamic, balanced-growth setting with re-contracting Calvo (1983). The resulting wage-rigidity framework dampens income effects in the short run, thus allowing significant responses of hours to aggregate shocks. In reduced form, the model dynamics are similar to that in Jaimovich and Rebelo (2009), with their habit parameter replaced by our probability of wage-contract resetting. That is, if wage contracts are reset frequently, labor supply behaves in accordance with King, Plosser and Rebelo (1988) preferences, whereas if they are never reset, we obtain the setting in Greenwood, Hercowitz and Huffman (1988).

A widely held view is that there is significant short-run wage rigidity and that this rigidity is an important element of the transmission mechanism of macroeconomic shocks.[1] There is so far no consensus, however, about the modeling of such wage rigidity. The applied quantitative-theory literature following Erceg, Henderson and Levin (2000) (see also Chapter 6 in Galí (2015)), for example, proceeds in analogy to the modeling of price rigidity in new-Keynesian settings: workers monopolistically choose an hourly wage at which they must supply whatever firms demand of their unique labor service. While both elegant and useful, the monopoly assumption appears quite strong for most workers.[2] Moreover, the restriction to a constant nominal hourly wage appears hard to square with actual work practices, in particular when a substantial rise in firms' demand moves workers far away from their supply curve. In this paper, we propose an alternative

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[1] Christiano, Eichenbaum and Evans (2005) argue that rigid wages is the key friction needed for quantitative new-Keynesian model to match empirical impulse responses to monetary policy shocks. Olivei and Tenreyro (2010) show that wage rigidities play an important role for the transmission of monetary policy. Wage rigidities also have stark distributional consequences, which greatly affect the dynamics of HANK models, see Broer et al. (2020).

[2] Monopolistic competition in wage setting may be interpreted as stemming from different occupations being organized in small unions, see Galí (2022). A wide body of recent empirical studies have, in contrast to the assumption in the EHL model, documented that labor markets are increasingly characterized by monopsonistic rather than monopolistic competition, see, e.g., Berger, Herkenhoff and Mongey (2022) and the literature discussed therein.

framework for studying wage rigidity. In particular, we follow Chari (1983) in describing the ex-ante wage-setting stage, in advance of observing macroeconomic shocks, as one of choosing a *wage contract*: a schedule of wage-hours pairs from which the firm, as shocks hit ex post, can choose one. Ex-ante, there is perfect competition with the result that firms offer the contract that maximizes workers' ex-ante utility. With this setting, hours worked are demand-determined ex post – firms have the "right to manage". Yet workers are not asked to work harder without extra compensation; in the Chari setup this compensation is in line with their optimal labor-leisure tradeoff.

We depart from Chari's setup in some distinct ways. First, because we focus on the fluctuations of hours, including on the individual level, we allow hours to be a continuous variable; in contrast, Chari's analysis was motivated by the implicit-contracts literature on employment vs. non-/un-employment.[3] Second, we adopt a preference specification that is consistent with balanced growth – from King, Plosser and Rebelo (1988) (KPR) – to maintain consistency with the applied macroeconomic literature. In particular, hours and consumption enter preferences additively, and consumption preferences have the log-form. Third, we assume that workers are fully insured; again, Chari's interest lay in implicit insurance aspects whereas we focus on the choice of wage-hours pairs and a comparison with standard representative-agent models. Fourth, we embed the Chari formulation in a dynamic model and assume re-contracting à la Calvo (1983).

We find that the resulting setting, while building on preferences with strong income effects, delivers high short-run intertemporal substitutability of labor. In fact, our model is very similar, in a reduced-form sense, to the setting proposed by Jaimovich and Rebelo (2009), who employ KPR preferences with habits, delivering zero income effects in the short run but significant income effects in the long run when habits have had time to adjust. The persistence of habits in their setting is replaced, in our model, with the probability of re-contracting (the Calvo parameter). Thus, if wage contracts are reset very frequently, labor supply behaves in accordance with KPR, whereas if they are rigid for a long time, labor supply behaves in accordance with Greenwood, Hercowitz and Huffman (1988), thus allowing significant responses in hours to aggregate shocks. The accompanying wage dynamics are, however, different from these settings. In our model, the cyclicality of the average real hourly wage is linked to the steady-state level of the labor share and, depending on the setup, the same response of hours to an underlying shock may be consistent with a procyclical, acyclical or countercyclical response of the average real hourly wage.

The details of the wage contract work as follows: the *marginal wage* is chosen so to make the contract ex-post efficient: the equilibrium amount of hours worked maximizes the joint surplus of the firm-worker pair. Thus, it equates the marginal rate of transformation with the marginal rate of substitution. In particular, it takes into account the increasing marginal disutility of hours worked. The *base wage* is chosen to be high enough to make workers agree to the contract. Because the contract is non-contingent on shocks, however, it

[3]See Rosen (1985) for a survey of the implicit-contracts literature.

only takes into account how the level of consumption responds to shocks on average. With aggregate shocks small in relation to firm-level shocks, aggregate shocks do not shift the labor supply curve.

With *nominally* rigid wages, our model delivers a wage Phillips curve that is qualitatively similar to that in EHL. Conceptually, however, the existence of a wage Phillips curve in our framework does not rely on worker market power nor worker wage setting, but is instead consistent with a perfectly competitive labor market at the contracting stage. Quantitatively, the implications are also different: our Phillips curve describes a tradeoff between inflation in marginal wages and real activity, as opposed to average wages and real activity.

In reduced form, our contracting model is consistent with the common assertion of limited income effects in the short run to explain aggregate dynamics of hours worked.[4] This does not mean that preferences with limited income effects are a substitute for a more realistic model of labor-market institutions that include rigid wage contracts. The reduced-form similarity only applies to the dynamics of hours worked, and not the dynamics of labor earnings. Moreover, the exact equivalence only holds in the particular benchmark case with complete asset markets, full labor divisibility and time-dependent recontracting. Going forward, a key area for future research is to explore how the addition of salient frictions changes the quantitative implications of rigid wage contracts for aggregate dynamics, as well as its implications for policy and welfare.

I The contracting problem

Consider a firm-worker pair that interacts for two periods. In the second period, the firm operates the production function $ZF(N)$, where N is labor input by the worker, and Z is a stochastic productivity term. In the first period, before productivity Z is known, the firm offers the worker a contract with the aim of maximizing expected profits $ZF(N) - W^s$ where W^s is total wage payments. A contract specifies a relation between wage payments W^s and labor input N and can be accepted or rejected by the worker. After productivity becomes known, the firm has the "right to manage": it can choose any combination of wage payments and labor input allowed by the contract.

The worker has separable preferences in wage payments and hours worked. If rejecting the contract, the worker receives reservation utility $\underline{U}$. If accepting the contract, the worker receives labor income W^s in exchange for N hours demanded by the firm after the shock is realized. Denote the disutility of hours worked with $v(N)$ and the value of receiving income W^s with $V(W^s)$. In this section, the worker has reduced-form preferences in wage payments; in an equilibrium model, the value of wage payments is derived from preferences over consumption and the financial market structure. In the case of financial autarky, $V(\cdot)$ is the consumption utility from directly consuming W^s.

[4] Both Jaimovich and Rebelo (2009) and Greenwood, Hercowitz and Huffman (1988) preferences have been used extensively in the quantitative macroeconomic literature, see, e.g., Kaplan, Moll and Violante (2018), Fukui, Nakamura and Steinsson (2021), Winberry (2021) and McKay and Wieland (2021) for recent applications.

Both the worker and the firm know the distribution of productivity, which has full support on $[0, \infty)$, at the contracting stage. Contracts are "rigid" in the following sense: first, the contract cannot be renegotiated in the second period. Second, wage payments can only depend on hours worked. There are no other restrictions on the contract.

The wage-hours schedule $W^s(N)$ can be specified in terms of a *marginal wage curve* $W(n)$ and a *base wage* W_{min}: $W^s(N) = \int_0^N W(n)dn + W_{min}$. The base wage, which is paid if zero hours of work is demanded, should not be confused with the expected wage $\mathbb{E}[W^s(N)]$.

Second period The firm's problem in the second period, given a wage schedule specified by $W(\cdot)$ and W_{min}, is given by

$$\max_N ZF(N) - \int_0^N W(n)dn - W_{min}. \tag{1}$$

For a standard production function, the optimal labor input chosen by the firm is such that marginal productivity equals the *marginal* wage, $ZF'(N) = W(N)$. Given the marginal wage function $W(\cdot)$, this optimality condition implicitly solves for hours as a function of productivity: $N = N(Z)$.

First period The firm's problem in the first period is to maximize expected profits subject to providing the worker her reservation utility:

$$\max_{W(\cdot), W_{min}, N(\cdot)} \mathbb{E}\left[ZF(N(Z)) - \int_0^{N(Z)} W(n)dn - W_{min}\right] \tag{2}$$

$$s.t. \;\; \mathbb{E}\left(V\left(\int_0^{N(Z)} W(n)dn + W_{min}\right) - v(N(Z))\right) \geq \underline{U}, \tag{3}$$

$$W(N(Z)) = ZF'(N(Z)). \tag{4}$$

By a substitution of variables, this program can recasted as a standard calculus-of-variations problem. In the Online Appendix, we show that the solution to this contracting problem is characterized by an ordinary differential equation. In general, this equation has to be solved using numerical methods. Proposition 1 characterizes the optimal contract for the case of a linear value function $V(\cdot)$ in wage payments. This case is particularly relevant for a wide class of macroeconomic models as it describes the contracting problem in an environment with idiosyncratic firm shocks where households can fully insure against fluctuations in individual labor income.

Proposition 1. *With $V'(\cdot) = \frac{1}{\xi}$, where ξ is a constant, the optimal contract sets the marginal wage equal to the marginal rate of substitution, $W(N) = \xi v'(N)$ and hours worked are given by $ZF'(N) = \xi v'(N)$.*

Proof. With linear $V(\cdot)$, utility is transferable between the worker and the firm. Therefore, the first best can be obtained as follows.

Solving for $\mathbb{E}\left[\int_0^{N(Z)} W(n)dn + W_{min}\right]$ in the constraint given by (3) and inserting it into the objective gives the objective (ignoring constants) $\mathbb{E}\left[ZF(N(Z)) - \xi v(N(Z))\right]$. The unconstrained maximum for this objective is obtained when $N(Z)$ satisfies $ZF'(N(Z)) = \xi v'(N(Z))$. To obtain this unconstrained maximum while satisfying the constraint given by (4), $W(\cdot)$ has to satisfy $W(N(Z)) = \xi v'(N(Z))$, i.e., $W(N) = \xi v'(N)$. Finally, W_{min} can be adjusted so that the constraint (3) holds. □

Proposition 1 states that hours worked will be efficient; the role of the contract is to make the firm internalize the worker's disutility of working more hours. It can easily be shown that this property does not depend on the particular setup of surplus splitting we have assumed here. For example, the same result follows if a union of workers, rather than the firms, offers the contract subject to a reservation profit level or if workers and firms bargain over the total surplus under a Nash bargaining protocol. Such alternative assumptions affect the level of base pay W_{min}, but the efficiency property follows directly from the assumption that utility is linear, implying that total surplus is maximized when the contract is ex-post efficient.

II Static equilibrium

We now embed the contracting problem in a static general-equilibrium environment with many firms and workers. We study the response of aggregate variables to changes in aggregate productivity A in the "long-run" (A is known before contracts are written) and the "short-run" (A is an unexpected shock realized after contracts are written). The results show how rigid wage contracts make business-cycle models consistent with large fluctuations in hours worked and small average-wage fluctuations in the short run while still maintaining the property of balanced growth in the long run.

There is a continuum of firms indexed by i owned collectively by workers through a diversified mutual fund. Firms match one-to-one with workers. Each firm operates the production function $Y_i = Z_i F(N) = Z_i N^{1-\alpha}$, with productivity $Z_i = A \times A_i$.[5] Firms and workers expect aggregate productivity A to be constant, normalized to 1. Idiosyncratic productivity A_i is equal to 1 on average, but subject to shocks that are i.i.d. across firms. The pool of firms is large, and there is free entry to posting contracts, implying that firms make zero profits in expectation.

There is a continuum of individual workers of unit mass. With slight abuse of notation and in anticipation of the equilibrium allocation, we index workers by the firm i with which they are matched. Worker utility

[5] We assume one-to-one matching and that labor is the only factor of production to simplify the exposition. The contracting problem between a firm and a worker is the same in the standard neoclassical setup with large firms employing constant-returns-to-scale production functions over total hours and capital, given that the capital stock and the number of workers are chosen before the realization of productivity.

is separable between consumption and hours worked, and consistent with balanced growth following King, Plosser and Rebelo (1988):

$$\begin{aligned} U(C,N) &= u(C) - v(N), \\ &= \log(C) - \kappa \frac{N^{1+\psi}}{1+\psi}. \end{aligned} \tag{5}$$

Before contracting, workers can trade a complete set of financial securities. In addition to labor income $W^s(N_i)$, worker i receives an exogenous endowment e_i. Endowment income is included to permit consideration, in a reduced-form way, of a balanced-growth setting where households also receive some non-labor income (e.g., income from capital) that grows proportionally to anticipated changes in TFP. Without loss of generality we set $e_i = e$ for all households. In addition, the workers also receive profits from their ownership of the firms (expected to equal 0 in equilibrium).

A worker's labor income is determined by the labor contract to which she has agreed. Complete markets imply that workers' marginal utility of consumption is independent of the idiosyncratic productivity shocks, and therefore, that the workers behave *as if* they belong to a representative family, for which the marginal utility of consumption is constant at the contracting stage (see the Online Appendix). The contracting problem in this environment is therefore the same as that considered in Proposition 1. However, the reservation utility $\underline{U}$ and the inverse level of marginal utility ξ are determined in equilibrium. In particular, free entry of firms implies that $\underline{U}$ adjusts so that expected profits are zero.

Definition 1. *A competitive equilibrium consists of a wage-hours schedule $W^s(N)$, an hours schedule $N(A_i)$, individual consumption C_i, and aggregate production Y such that*

- *given the worker's inverse marginal utility of consumption ξ, $W^s(N)$ solves the contracting problem,*
- *the reservation utility $\underline{U}$ is such that $\mathbb{E}\left[Z_i F(N_i) - W^s(N_i)\right] = 0$,*
- *ex-post hours for worker i, $N_i = N(Z_i)$, satisfy firm optimality given the contract $W^s(N_i)$ and realized productivity Z_i,*
- *the goods market clears: $C_i = C = Y + e$ for all i with $Y = \int_{i=0}^{1} Z_i F(N_i) di$,*
- *and the inverse marginal utility of consumption is $\xi = \frac{1}{u'(C)} = C$.*

Computing the equilibrium Proposition 1 characterizes the marginal wage schedule $W(N)$ given inverse marginal utility ξ and the response of hours to idiosyncratic productivity shocks: $N_i = N_i(A_i; \xi)$. Given the marginal wage schedule, the zero-profit condition pins down the base wage W_{min}. In equilibrium, inverse marginal utility of consumption is given by $\xi = Y + e$, so we write $N_i = N_i(Z_i; Y + e)$. Equilibrium aggregate production Y is solved from the market-clearing condition $Y = \int_i Z_i F(N_i(Z_i; Y + e)) di$.

The response to changes in expected productivity Consider a change in aggregate productivity from A to A' that is fully anticipated at the contracting stage. On top of increasing the productivity of matches, we also assume that this shock scales households' endowment income: $e' = e \times A'$. Although our economy is static, this experiment corresponds to a balanced-growth path where all household income grows at the same rate. Proposition 2 establishes that in response to such a change, hours worked are not affected. The fact that firms and workers agree to the wage-hours contract ex ante, as opposed to hours being determined in a spot market ex post, does not change the balanced-growth property of the KPR preference specification (5).

Proposition 2. *In response to a change in aggregate productivity that is anticipated in the contracting period, total hours are unchanged, and output moves one-for-one with productivity,*

$$Y' = A'Y,$$
$$N' = N.$$

Proof. By Proposition 1, hours worked in each contract i is given by $Z_i F'(N_i) = \xi v'(N_i)$, where inverse marginal utility ξ of consumption is given by $\xi = Y + e$ and total output is given by $Y = \int_0^1 Z_i F(N_i) di$. Equilibrium hours are thus characterized by

$$Z_i F'(N_i) = \left(\int_0^1 Z_j F(N_j) dj + e \right) v'(N_i).$$

With a shift in aggregate productivity such that $Z_i' = A' Z_i$, the new equilibrium is characterized by

$$A' Z_i F'(N_i) = \left(\int_0^1 A' Z_j F(N_j) dj + A' e \right) v'(N_i)$$

and it is readily seen that equilibrium hours do not depend on the aggregate productivity level A'. □

Given that output scales with aggregate productivity and hours are unchanged, the zero-profit condition implies that total wage payments W^s and the average wage $\frac{W^s}{N}$ scale with aggregate productivity as well.

The response to changes in unexpected productivity We now consider the response to an unexpected increase in aggregate productivity from A to A' (an "MIT" shock).

Proposition 3. *In response to an aggregate productivity shock that is unexpected at the contracting stage,* *total hours and total output respond by*

$$Y' = (A')^{1+(1-\alpha)/(\alpha+\psi)} Y > AY,$$
$$N' = (A')^{1/(\alpha+\psi)} N > N.$$

Proof. At the match level, hours are given by $A'A_iF'(N_i') = \xi v'(N_i')$. With $F(N) = N^{1-\alpha}$ and $v(N) = \kappa \frac{N^{1+\psi}}{1+\psi}$, we have $(1-\alpha)A'A_i(N_i')^{-\alpha} = \xi\kappa(N_i')^{\psi}$ or rearranging,

$$N_i' = \left(\frac{1-\alpha}{\xi\kappa}A'A_i\right)^{1/(\alpha+\psi)} = (A')^{1/(\alpha+\psi)}N_i.$$

Inserting hours worked into the production function yields $Y_i' = (A')^{1+(1-\alpha)/(\alpha+\psi)} Y_i$ and aggregating over the matches yields the proposition. □

In Proposition 2, the contract was conditioned on the expected increase in productivity, while in Proposition 3, it was not. In response to the unexpected shock, firms find it optimal to increase hours worked. This is achieved by raising the marginal wage payment, in line with the contract. The fact that the contract is not conditioned on the shock, however, means that it does not take into account that the equilibrium increase in consumption of the workers also diminishes their marginal utility of consumption. Contracts written before aggregate shocks are realized thus "turn off" the income effect that otherwise offsets the substitution effect in response to anticipated shocks in Proposition 2.

We now consider the response of the average hourly wage $\bar{W}_t = \frac{W^s}{N}$ to the unexpected shock. Proposition 4 shows that the cyclicality of the average wage is determined by the output elasticity of the production function and the labor share.

Proposition 4. *In response to an unexpected productivity shock, the equilibrium elasticity of the average wage with respect to hours, $\epsilon_{\bar{W},N}$, is given by*

$$\epsilon_{\bar{W},N} = \frac{1-\alpha}{LS} - 1$$

where $LS = \frac{W^s}{Y}$ is the steady-state labor share of income.

Proof. With $\bar{W} = \frac{W^s}{N}$, the first-order response of the average wage is given by (using differentials)

$$\frac{\mathrm{d}\bar{W}}{\bar{W}} = \frac{\mathrm{d}W^s}{W^s} - \frac{\mathrm{d}N}{N}.$$

Since wage payments for a match i is given by $W^s(N_i) = \int_0^{N_i} W(n)dn + W_{min}$, we have

$$\mathrm{d}W^s(N_i) = W(N_i)\mathrm{d}N_i.$$

Firm optimality implies $W(N_i) = Z_i F'(N_i)$. We thus have $W(N_i)\mathrm{d}N_i = \frac{F'(N_i)N_i}{F(N_i)} Z_i F(N_i) \frac{\mathrm{d}N_i}{N_i} = (1 - \alpha)Y_i \frac{\mathrm{d}N_i}{N_i}$. From the proof of Proposition 3, the hours response to an aggregate productivity shock is equal across matches, $\frac{\mathrm{d}N_i}{N_i} = \frac{\mathrm{d}N}{N} = \frac{\mathrm{d}A}{\alpha+\psi}$. We therefore arrive at $\mathrm{d}W^s(N_i) = (1-\alpha)Y_i \frac{\mathrm{d}N}{N}$ and, integrating over all matches, we get,

$$\mathrm{d}W^s = (1-\alpha)Y \frac{\mathrm{d}N}{N}.$$

Finally, this together with $LS = \frac{W^s}{Y}$ gives us

$$\frac{\mathrm{d}\bar{W}}{\bar{W}} = \left(\frac{1-\alpha}{LS} - 1 \right) \frac{\mathrm{d}N}{N}.$$

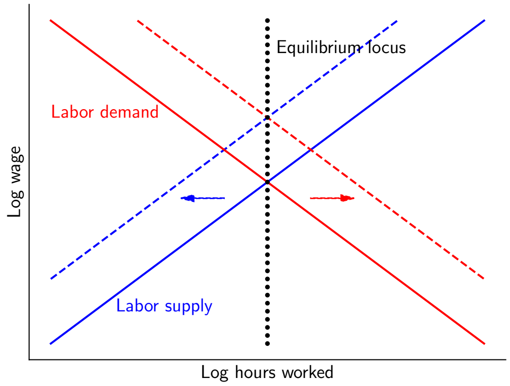
□

Proposition 4 reflects that firm optimality, $W(N_i) = Z_i F'(N_i)$, restricts the cyclicality of the marginal wage with respect to hours. Given this cyclicality, a higher steady-state labor share reduces the comovement between *average* wages and output. Under our benchmark assumptions, steady-state profits are zero, implying $LS = 1$. The elasticity of output to labor inputs equals $1 - \alpha$ and we thus have $\epsilon_{\bar{W},N} < 0$: average wages are countercyclical.

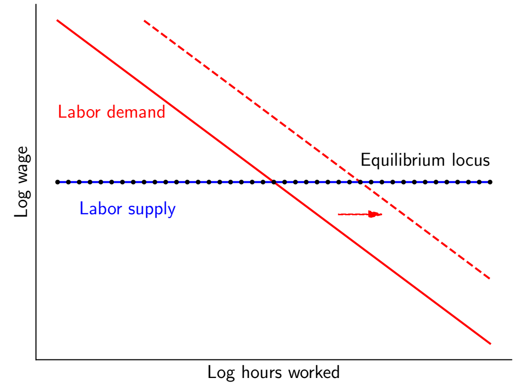
More generally, the zero-profit condition implied by free entry of firms is but one way to embed our contracting model in a general-equilibrium environment. The same contracting problem can be included in, e.g., a competitive model with capital or a frictional labor market. At the contracting stage, such alternative frameworks only change the equilibrium labor share. In particular, one can show that when production uses capital (predetermined at the contracting stage) in addition to labor in a Cobb-Douglas production function, $Y = AK^\alpha L^{1-\alpha}$, the labor share is $LS = 1-\alpha$, and by Proposition 4, the average wage is acyclical. Similarly, in a frictional labor market environment à la Diamond-Mortensen-Pissarides, with linear production ($\alpha = 0$) and vacancy posting costs, steady-state firm profits are positive and the average hourly wage of existing matches is procyclical.

Comparisons Figure 1 displays the short-run behavior of hours worked in response to unexpected changes in aggregate productivity for our rigid-contracts model alongside three comparison models.

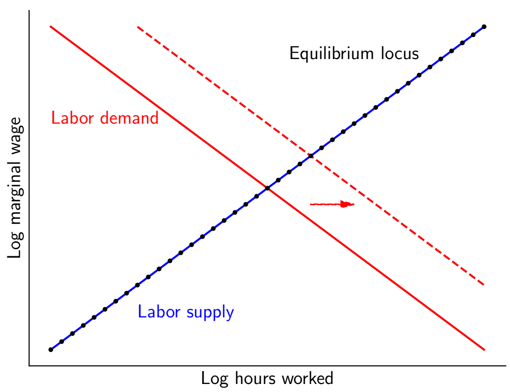
In Subfigure 1a, we display the Marshallian cross in a neoclassical spot labor market. The labor demand



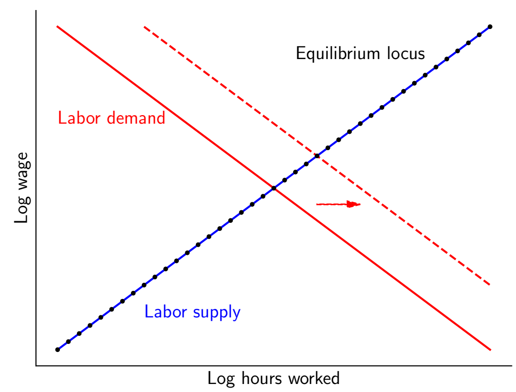
(a) Neoclassical spot market.



(b) Rigid wages.



(c) Rigid wage contracts.



(d) Neoclassical spot market with GHH preferences.

Figure 1: The Marshallian cross in four types of labor markets.

curve is given by the F.O.C. of the firm's optimality problem, which, when log-linearized, is

$$\log w = \log(1-\alpha) + a - \alpha \log n. \tag{6}$$

The supply curve is given by the household's intratemporal optimality condition, which with preferences as in Equation (5) is given by

$$\log w = c + \psi \log n. \tag{7}$$

In response to a positive shock to aggregate productivity a, the demand curve shifts out. With an upward-sloping supply curve, hours worked and the wage level increase in partial equilibrium. In general equilibrium, the income effect from the increase in household consumption shifts the supply curve inward. The balanced-growth property of preferences (5) implies that the partial-equilibrium substitution effect and the general-equilibrium income effect cancel, and the equilibrium locus in response to an aggregate productivity shock is formed by a vertical line.

In Subfigure 1b, we display the Marshallian cross in a labor market with rigid, or predetermined, hourly wages, where firms unilaterally determine hours in response to shocks. This is the labor-market setup in the Erceg, Henderson and Levin (2000) (EHL) model, widely used in the new-Keynesian literature (we discuss this model further in Section III). Here, the labor demand curve is the same, but the short-run supply curve is now a flat line: workers have to accept any quantity of hours demanded by the firm ex post. In response to a positive shock to aggregate productivity a, the demand curve shifts out and, since the supply curve is flat, there is a larger partial-equilibrium increase in hours worked compared to the neoclassical spot market. Moreover, with the wage contract not being contingent on the shock, there is no income effect in general equilibrium. The supply curve does not shift, and the equilibrium locus coincides with the flat supply curve.

In Subfigure 1c, we display the Marshallian cross in our model, with unrestricted wage-hours contracts. The labor demand curve is the same, and the optimal contract prescribes that the supply curve is also given by the worker optimality condition (7). However, w in (7) is the marginal wage rather than the total wage. In the neoclassical spot market and the rigid-wage model, the marginal wage and the average wage coincide; in our setup, they do not. As in the rigid-wage model, the contract in our model is not contingent on aggregate shocks so the supply curve does not respond to the aggregate shock. In effect, the general-equilibrium response to the productivity shock in our model mimics the partial-equilibrium response in the neoclassical spot market, without any feedback from the increase in income on consumption utility.

Therefore, the equilibrium response to unexpected productivity shocks in our model is identical to that in an alternative environment with a neoclassical spot market for labor with preferences featuring no income effects. A class of such preferences was proposed by Greenwood, Hercowitz and Huffman (1988) (GHH):

$$U(C,N) = u(C - v(N)). \tag{8}$$

With GHH preferences, the optimality condition in the neoclassical spot market is $AF'(N) = v'(N)$, just as in our contractual setup. We summarize this result in Proposition 5.

Proposition 5. *The response of output and hours to an unexpected shock to aggregate productivity in our rigid-contracts model is identical to that in an alternative environment where hours worked are determined in a competitive spot market but where worker preferences are given by* (8)*.*

The Marshallian cross with GHH preferences is displayed in Subfigure 1d. GHH preferences are popular in business-cycle analysis, as they give rise to larger equilibrium fluctuations in labor inputs in response to short-run shocks when embedded in a standard spot labor market. Because of the absence of income effects on labor supply, they are not, however, consistent with long-run balanced growth.[6] Our contractual setup, in contrast, features both balanced growth (in response to "long-run", or anticipated, changes in productivity) and no income effects in the short run.

Remarks We have considered a deterministic path of the economy in response to a one-time unexpected ("MIT") shock to productivity. In general, workers and firms may expect that aggregate productivity is drawn from some known distribution. The rationale for considering MIT shocks is that aggregate shocks are small relative to idiosyncratic shocks in most settings. In the limit of arbitrarily small aggregate shocks, certainty equivalence holds and the response to an unexpected MIT shock is a sufficient statistic for simulating the model with stochastic shocks. If aggregate shocks are large relative to the idiosyncratic shocks, then workers require compensation for the unconditional correlation between consumption and hours, increasing the importance of income effects. The contracting problem (2) then needs to be adapted so that firms discount profits using households' stochastic discount factor. In the limit with *only* aggregate productivity shocks and no idiosyncratic shocks, workers' marginal utility of consumption is perfectly negatively correlated with firm productivity. Moreover, with balanced-growth preferences and no endowment income, $e = 0$, their ratio is constant. The Online Appendix shows that the optimal contract then implements constant hours and wage payments independent of productivity.

We have assumed that asset markets are complete. Maintaining the rigidity assumption, we conjecture that other assumptions regarding market structure will change the slope of the supply curve in Figure 1c, but that the supply curve is unresponsive to the shock. For example, if embedding the contracting problem in an Aiyagari (1994)-style model of limited consumption insurance, we conjecture that the optimal contract

[6]GHH preferences have also been criticized for implying a strong complementarity between consumption and leisure that may generate implausibly large short-run responses to fiscal shocks (Auclert, Bardóczy and Rognlie, 2021).

partially insures worker's consumption, and the shape of the supply curve will depend on the degree of risk aversion and the distribution of the idiosyncratic shocks.

III Dynamic equilibrium

The previous section presented a static model with fully rigid contracts, which we used to interpret the effects of contract rigidity on macroeconomics dynamics in the short vs. long-run. In this section, we embed our contracting problem in an infinite-horizon model with Calvo (1983) rigidity: each contract can be rewritten with a constant probability every period. Workers have per-period preferences given by (5) and seek to maximize expected discounted utility with a constant time discount factor β. We maintain the assumptions that there is no capital, firms are owned by workers through a diversified mutual fund and that asset markets are complete (implying aggregation to a representative worker). Let $\beta^t \Lambda_t$ denote the discount factor of the representative worker. For simplicity, we set the endowment e and the net supply of assets to zero.

In the previous section we focused on real shocks to aggregate productivity, implicitly setting the price of consumption goods to a constant, normalized to 1. In this section we explicitly introduce a nominal price of consumption goods P, which allows us to also consider shocks to the price level. We stipulate that contracts are such that the wage-hour schedule cannot condition on any future aggregate shocks and that it is specified in nominal terms, i.e., the contract is *nominally rigid*.

As before, optimal firm behavior prescribes setting marginal nominal productivity equal to the marginal wage prescribed by the contract, $P_t A_t A_{i,t} F'(N_t) = W(N_t)$ where $W(N_t)$ is now the *nominal* wage-hours schedule. The problem of firms that can change their contract in period 0 is to offer an optimal nominal wage-hours schedule, knowing that the contract will be in force in period $t > 0$ with probability θ^t, where θ is the constant probability of the contract surviving to the next period.

We concentrate on settings where aggregate shocks are small relative to idiosyncratic shocks and accordingly solve the model by linearizing with respect to aggregate variables, implying that certainty equivalence holds. We therefore consider the firm's problem under a perfect-foresight path for the aggregate variables: firms take the path of aggregate productivity A_t, the price level P_t, and the discount factor $\beta^t \Lambda_t$ as given. The problem of a re-contracting firm at time 0 is thus

$$\max_{W(\cdot), W_{min}} \sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_t \mathbb{E}\left[A_t A_{i,t} F(N(P_t A_t A_{i,t})) - \frac{\int_0^{N(P_t A_t A_{i,t})} W(n) dn + W_{min}}{P_t}\right], \tag{9}$$

$$s.t. \quad \sum_{t=0}^{\infty} (\beta\theta)^t \Lambda_t \mathbb{E}\left[\left(\frac{\int_0^{N(P_t A_t A_{i,t})} W(n) dn + W_{min}}{P_t}\right) - v(N(P_t A_t A_{i,t}))\right] \geq \underline{U}, \tag{10}$$

$$P_t A_t A_{i,t} F'(N_{i,t}) = W(N_{i,t}). \tag{11}$$

Given the marginal wage schedule $W(\cdot)$, the constraint (11) implicitly defines $N_{i,t} = N(Z_{i,t})$, where $Z_{i,t} = P_t A_t A_{i,t}$ now includes the aggregate price level (that was implicitly normalised to 1 before). Since the contract takes the discount factor $\beta^t \Lambda_t$ as given, utility is again transferable as before. The solution to the firm's problem amounts to selecting $N(\cdot)$ such that joint surplus is maximized:

$$\max_{N(\cdot)} \sum_{t=0}^{\infty} \frac{(\beta\theta)^t \Lambda_t}{P_t} \mathbb{E}\left[Z_{i,t} F(N(Z_{i,t})) - \frac{P_t}{\Lambda_t} v(N(Z_{i,t}))\right], \tag{12}$$

and adjusting W_{min} so that the worker accepts the contract. The solution is characterized by a first-order condition that sets average expected marginal productivity equal to the average expected marginal rate of substitution, see the Online Appendix. Log-linearizing this optimality condition, the log deviation of $N(Z)$ is given by

$$\hat{n}(Z) \quad = \quad -\frac{1}{\alpha + \psi}(1 - \beta\theta)\mathbb{E}\sum_{t=0}^{\infty}(\beta\theta)^t \left(\hat{p}_t - \hat{\lambda}_t\right) \tag{13}$$

where $\hat{x}_t$ represents the log deviation of any variable X_t from its value in the absence of aggregate shocks. Write $\hat{\xi}_t = (1 - \beta\theta)\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k(\hat{p}_{t+k} - \hat{\lambda}_{t+k})\right]$ for the "expected average price of consumption utility". The corresponding wage-hours schedule is given by

$$W(N_{i,t}) = (1 + \hat{\xi}_0)\kappa N_{i,t}^{\psi}.$$

The slope of the optimal contract is set such that the contract matches the marginal rate of substitution for the worker on average. As before, the base wage W_{min} can be calculated but does not affect the determination of hours worked.

Aggregation The Calvo structure of contract duration implies, to a first-order approximation, simple dynamic relationships between labor-market aggregates when aggregating across contract vintages. A vintage t is associated with the slope of its nominal marginal-wage curve, $\hat{\xi}_t$. Within a contract cohort t, idiosyncratic productivities $a_{i,t}$ average out, and hours in period $t + k$, $\hat{n}_{t+k|t}$, are given by

$$\hat{a}_{t+k} + \hat{p}_{t+k} = \hat{\xi}_t + (\psi + \alpha)\hat{n}_{t+k|t}.$$

At time t, a share $(1 - \theta)\theta^s$ of the population has a contract of vintage $t - s$ so aggregate hours worked are given by

$$\hat{n}_t = \frac{1}{\psi + \alpha}\hat{a}_t - \frac{1}{\psi + \alpha}(1 - \theta)\sum_{s=0}^{\infty}\theta^s(\hat{\xi}_{t-s} - \hat{p}_t). \tag{14}$$

Define the backward-looking sum of the contract vintages $\hat{w}_t^{all} = (1-\theta)\sum_{s=0}^{\infty}\theta^s\hat{\xi}_{t-s}$. The object $\hat{w}_t^{all}$, the across-vintages average of the slopes of the marginal wage schedules, is like an allocative nominal wage: Equation (14) can be interpreted as a demand curve in a Marshallian diagram like those considered in Figure 1.

Define the *allocative real wage* as $\hat{\omega}_t^{all} = \hat{w}_t^{all} - \hat{p}_t$. In the Online Appendix, we show that Equation (14) can be rewritten like a forward-looking Phillips curve relating the growth rate in the nominal allocative wage $\pi_t^{all} = w_t^{all} - w_{t-1}^{all}$ to the current level of the real allocative wage. Together with the firm-optimality condition that determines hours ex post, we have

$$\pi_t^{all} = \beta\mathbb{E}_t\pi_{t+1}^{all} - \gamma(\hat{\omega}_t^{all} + \hat{\lambda}_t), \tag{15}$$

$$\hat{\omega}_t^{all} + \psi\hat{n}_t = \hat{a}_t - \alpha\hat{n}_t, \tag{16}$$

with $\gamma = \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Put together, the growth rate in the nominal allocative wage is related to the deviation of marginal productivity, $\hat{a}_t - \alpha\hat{n}_t$, from the marginal rate of substitution, $-\hat{\lambda}_t + \psi\hat{n}_t$. We arrive at the following proposition:

Proposition 6. *Taking goods-price inflation π_t, marginal consumption utility $\hat{\lambda}_t$ and the initial real allocative wage $\hat{\omega}_{-1}$ as given, the labor-market equilibrium $\{\hat{n}_t, \hat{\omega}_t, \pi_t^{all}\}$ is summarized by a wage Phillips curve*

$$\pi_t^{all} = \beta\mathbb{E}_t\pi_{t+1}^{all} - \gamma(\hat{a}_t + \hat{\lambda}_t - (\alpha+\psi)\hat{n}_t), \tag{17}$$

a firm-optimality condition,

$$\hat{\omega}_t^{all} + \psi\hat{n}_t = \hat{a}_t - \alpha\hat{n}_t, \tag{18}$$

and an accounting identity

$$\hat{\omega}_t^{all} = \hat{\omega}_{t-1}^{all} + \pi_t^{all} - \pi_t. \tag{19}$$

Here, we have considered the labor-market equilibrium in isolation, i.e., π_t and $\hat{\lambda}_t$ are exogenous. To close the general equilibrium, we need to add equations describing the dynamics of consumption and inflation. For example, integrated in the textbook new-Keynesian model, aggregate consumption and inflation obey the usual IS curve and a Taylor rule for monetary policy.

The dynamics of rigid real contracts are obtained by setting $\pi_t = 0$. Then π_t^{all} represents the growth rate of the real allocative wage.

Comparisons Our dynamic model of rigid wage contracts implies equilibrium dynamics that are qualitatively similar to two other labor-market models widely used for business-cycle analysis: the spot-market model with habit preferences introduced by Jaimovich and Rebelo (2009) (JR) and the monopolistic model of nominal wage rigidity introduced by EHL.

JR considered a neoclassical spot labor market in which workers have a per-period utility function:

$$U(C_{i,t}, N_{i,t}, X_{i,t}) = \frac{\left(C_{i,t} - \frac{\kappa N_{i,t}^{1+\psi} X_{i,t}}{1+\psi}\right)^{1-\sigma} - 1}{1-\sigma} \tag{20}$$

where $X_{i,t}$ represents a habit. When habits are determined by aggregate consumption and thus external to individual workers, $X_{i,t} = X_t$ and evolves according to[7]

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}.$$

JR's preference specification nests GHH preferences, discussed in the previous section, with $\gamma = 0$. In contrast to GHH, with $\gamma \in (0, 1]$, optimal household choice is consistent with long-run balanced growth.

With JR preferences, the marginal rate of substitution between consumption and leisure is given by $MRS_{i,t} = \kappa N_{i,t}^{\psi} X_t$. With a spot market for labor, marginal productivity equals this marginal rate of substitution every period, which, when log-linearized, provide the equilibrium conditions

$$\hat{x}_t = \gamma \hat{c}_t + (1-\gamma)\hat{x}_{t-1}, \tag{21}$$

$$\hat{x}_t + \psi \hat{n}_t = \hat{a}_t - \alpha \hat{n}_t. \tag{22}$$

Equations (21) and (22) compare to Equations (15) and (16) in our model. Both models feature a short-run wedge between the marginal rate of substitution and the marginal rate of transformation between hours and consumption goods as both $\hat{x}_t$ and $\hat{\omega}_t^{all}$ adjust slowly. For JR preferences, this wedge results from a persistent habit that depends on past aggregate consumption $\hat{c}_t$. For our rigid-contracts model, the wedge results from past expectations of the aggregate discount factor $-\hat{\lambda}_t$. In general equilibrium with KPR preferences, we typically have $-\hat{\lambda}_t = \hat{c}_t$. In the long-run, the marginal rate of substitution equals the marginal rate of transformation, consistent with balanced growth in both models. The rigid-contract model thus shares the three essential properties that JR sought when introducing the preferences (20): i) limited income effects in the short run, ii) convergence to balanced growth in the long run and iii) a parameter controls the speed of convergence to balanced growth. With JR preferences, the parameter is the degree of habit formation γ in (21); in our model, it is the Calvo contract duration parameter θ.

[7] JR's original preference specification had internal rather than external habits. This distinction matters for the welfare properties of the model, but is immaterial for the results we discuss here.

EHL considered a model with Calvo-style rigid wage setting and complete asset markets, similar to our model. In their setup, firms do not offer optimal contracts in a competitive market. Rather, workers are monopolistic suppliers of a differentiated labor input, subject to a CES demand curve, and constrained to set a constant nominal hourly wage for the duration of the contract. Ex post, workers are committed to providing whatever hours are demanded by the firm at this wage level. Once the wage is allowed to be reset, the worker sets a new nominal wage to minimize the expected future distance between the real average wage and a frictionless optimal markup over her marginal rate of substitution. Assuming the preference specification (5) and that firms sell their goods in a competitive market, the EHL labor-market equilibrium is summarized by

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w - \gamma^{EHL}(\hat{\omega}_t - (\hat{c}_t + \psi \hat{n}_t)), \tag{23}$$

$$\hat{\omega}_t = \hat{a}_t - \alpha \hat{n}_t, \tag{24}$$

$$\hat{\omega}_t = \hat{\omega}_{t-1} + \pi_t^w - \pi_t, \tag{25}$$

with $\gamma^{EHL} = \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\delta\psi)}$. Here π_t^w is nominal inflation of average wages, $\hat{\omega}_t$ is the real average wage and δ is the slope of the CES demand curve. Equations (23)-(25) compare to Equations (17)-(19) in our model, for which typical general-equilibrium structures also imply $-\hat{\lambda}_t = \hat{c}_t$. Given a path of inflation π_t and aggregate consumption $\hat{c}_t$, both models describe a dynamic trade-off between a measure of nominal wage inflation and real activity. In EHL, however, the wage inflation measure is in terms of the average wage level, whereas in our model, the measure is in terms of an average of marginal wages. Moreover, the models differ in the slopes of the two Phillips curves, γ and γ^{EHL} (as γ^{EHL} increases with worker market power that plays no role in our competitive contractual setup); and the labor-demand equations ((18) and (24)), where our model takes account of the optimal labor-leisure tradeoff as specified in the contracts. With infinite Frisch elasticity ($\psi = 0$), the dynamics for hours worked in our setup are identical to those in EHL, as the labor supply curve in the Marshallian cross of our model is horizontal, just as in Subfigure 1b.

EHL's model has been criticized for assuming that workers must supply *any* amount of hours demanded. In a typical calibration, this setup may move workers far away from their labor supply curve without any possibility of exiting the relationship (Huo and Ríos-Rull, 2020). In our model, in contrast, workers are compensated by higher wages whenever they provide more hours. This reduces the distance between the hours required and their willingness to work.

The comparison of our model, with ex-ante competition and wage contracts offered by firms, to EHL also demonstrates that the EHL assumptions of worker market power and worker wage setting are not essential for generating a wage Phillips curve. Rather, the existence of a wage Phillips curve hinges on contracts that cannot be continuously renegotiated and hours worked that are determined by firm demand in response to shocks. These elements are consistent with a labor market that is perfectly competitive at the contracting stage.

IV Conclusion

The key assumptions of our contracting model are that firms have the right to manage ex post and that contracts are rigid: contracts cannot condition on shocks and cannot always be renegotiated in response to shocks. Our model implies short-run fluctuations in hours worked similar to those predicted by a spot labor market and GHH preferences. In contrast to such a setup, our model is consistent with balanced growth in the long run, similar to JR's generalization of GHH, but without assuming consumption habits. The implications for wage dynamics are, however, different. Depending on the model setup, average real wages may be procyclical, acyclical, or countercyclical. With shocks to the price level, the wage contracts generate a Phillips curve that is qualitatively similar to that in EHL. In contrast, however, our Phillips curve prescribes a tradeoff between inflation in marginal wages and real activity, as opposed to average wages and real activity, and its slope does not depend on the extent of worker market power.

The empirical support for contract rigidity is extensive, see, e.g., Grigsby, Hurst and Yildirmaz (2021), Hazell and Taska (2020) and Barattieri, Basu and Gottschalk (2014). Less is known about variations in intensive-margin hours worked and labor compensation in response to variation in labor demand within contracts. Using U.S. administrative data, Grigsby, Hurst and Yildirmaz (2021) report that cyclical fluctuations in bonuses and overtime compensation are small, suggesting that wage contracts are steep with respect to aggregate shocks (with a vertical labor supply curve in Figure 1c, hours worked and total compensation do not respond to a shift in labor demand, although the marginal wage does). Taken at face value, this suggests that there are other salient frictions, for example labor indivisibility, that increase the slope of the contracted labor supply curve relative to the simple benchmark model we have outlined here. However, more micro-level evidence is needed, as compensation for variable hours worked and effort may also appear in other forms, e.g., future wage increases and promotions. We regard further exploration of micro-level data on wage contracts, along with quantitative explorations of model extensions needed to match the data, as key areas for future research.

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