Macroeconomic dynamics with rigid wage contracts

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Rigid wage contracts: a research agenda

Several ongoing projects:

1. Macroeconomic dynamics with rigid wage contracts
   - Broer, Harmenberg, Krusell and Öberg, AER: Insights, forthcoming.
2. Rigid wage contracts and incomplete asset markets
3. Rigid wage contracts in frictional labor markets
4. Rigid wage contracts: estimation and implications using Norwegian micro data

Collaborators: Tobias Broer, Caio Koslyk, Per Krusell, Erik Öberg, Maria Olsson

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Background

- Consensus: wage rigidities fundamental for business-cycle fluctuations
  - Olivei-Tenreyro (2010), Carlsson-Björklund (Olsson)-Skans (2019): rigid wage setting key for transmission of monetary shocks
  - Broer-Hansen-Krusell-Öberg (2021), Auclert-Bardóczy-Rognlie (2021): transmission mechanism in monetary models without rigid wage setting makes little sense
  - Christiano-Eichenbaum-Evans (2005): rigid wage setting key assumption for quantitative models to match emprical IRFs
- No consensus: how to model rigid wages
- Approach in quant-macro literature: Erceg-Henderson-Levin (2000) (EHL)
  - Analogous to New-Keynesian price setting: workers set their own wage and firms choose hours worked
  - Key assumptions: (i) workers have monopoly power, (ii) the nominal hourly wage is fixed
  - Elegant and useful, but difficult to take to the data

This paper

- Goal: model of wage setting which makes sense at the micro level, to ultimately make macro models speak with micro data

- Today: a first step in this direction, establishing a theoretical baseline of optimal rigid wage contracts

- Key assumptions:
  1. Wage contracts are rigid: cannot condition on aggregate shocks, cannot be renegotiated with certainty
  2. Firm has the "right to manage": after the realization of shocks, the firm decides how many hours to extract given the contract
  3. Optimal contract features overtime pay

- Everything else is standard and frictionless: competitive labor market, fully divisible labor, complete asset markets, separable preferences etc.
  - Contracting problem similar to Chari (1983)

Insights

Optimal rigid contracts weaken income effects but preserve substitution effects in labor-leisure tradeoff

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- With full rigidity, our model generates hours responses *as if* in a spot market where workers have Greenwood-Hercowitz-Huffman (1988) preferences
  - With a Calvo (1983) rigidity, the model response is similar to a spot-market setting with Jaimovich-Rebelo (2009) preferences

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- With nominally rigid contracts, the model generates a Phillips curve
  - ...but without monopolistic competition or worker wage setting

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  - With a Calvo (1983) rigidity, the model response is similar to a spot-market setting with Jaimovich-Rebelo (2009) preferences
- With nominally rigid contracts, the model generates a Phillips curve
  - ...but without monopolistic competition or worker wage setting
- The *marginal wage* is allocative, the *average wage* is not; the same response in hours can be consistent with pro-, a-, or countercyclical fluctuations in the average wage

The contracting problem

General equilibrium

- Comparisons with other models

Dynamic model

- Comparisons with other models

Conclusion/going forward

- Rigid wage contracts with incomplete asset markets

The contracting problem

Environment

- Two periods: 1) contracting period, 2) production period
- The firm has a production function $Y = AF(N)$ and wants to maximize profits
- The level of productivity is ex-ante uncertain but the distribution is known
- The firm offers a wage schedule $W^s(N)$ to the worker
- The worker has preferences

$$U(W^s, N) = u(W^s) - v(N)$$

over wage payments W^s and hours worked N

The contract

- Period 1: the firm offers the worker a wage-hours schedule

$$W^s(N) = \underbrace{\int_0^N \underbrace{W(n)}_{\text{marginal wage}} \, \mathrm{d}n}_{\text{“variable pay”}} + \underbrace{W_{min}}_{\text{“base pay”}}$$

- The worker accepts the contract if it, in expectation, gives reservation utility $\underline{U}$

- Note: the contract is *incomplete*, cannot be conditioned directly on shock

- Period 2: Productivity A is realized and the firm unilaterally decides on hours worked

The contracting problem

- Period 2: Contract is given, the firm equalizes marginal production with marginal pay:

$$AF'(N) = W(N)$$

$\Rightarrow$ hours worked $N = N(A)$, implicitly given by $AF'(N(A)) = W(N(A))$

- Period 1: Maximize expected profits subject to worker's reservation utility and second-period optimality:

$$\max_{W(\cdot), W_{min}, N(\cdot)} \mathbb{E}\left[AF(N(A)) - \int_0^{N(A)} W(n)\,dn - W_{min}\right]$$
$$\text{s.t. } \mathbb{E}\left(u\left(\int_0^{N(A)} W(n)\,dn + W_{min}\right) - v(N(A))\right) \geq \underline{U},$$
$$W(N(A)) = AF'(N(A)).$$

The contracting problem: linear utility

- Optimization problem: choosing a function, not a variable
  - General solution can be characterized using standard tools from calculus of variations (see the paper)
- An interesting special case: linear consumption utility $u(W^s) = \frac{W^s}{\xi}$
  - Corresponds to an equilibrium with full insurance
- Contracting problem becomes

$$\max_{W(\cdot), W_{min}, N(\cdot)} \mathbb{E}\left[AF(N(A)) - \int_0^{N(A)} W(n)dn - W_{min}\right]$$

$$\text{s.t. } \mathbb{E}\left(\frac{1}{\xi}\left(\int_0^{N(A)} W(n)dn + W_{min}\right) - v(N(A))\right) \geq \underline{U},$$

$$W(N(A)) = AF'(N(A)).$$

The contracting problem: solution

With linear utility, the optimal contract maximizes total surplus

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$$\max_{W(\cdot), W_{min}, N(\cdot)} \mathbb{E}\left[AF(N(A))\right] - \mathbb{E}\left[\int_0^{N(A)} W(n)dn + W_{min}\right]$$

$$\text{s.t.} \quad \frac{1}{\xi}\mathbb{E}\left(\int_0^{N(A)} W(n)dn + W_{min}\right) - \mathbb{E}v(N(A)) = \underline{U},$$

$$W(N(A)) = AF'(N(A)).$$

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$$\text{s.t. } \mathbb{E}\left(\int_0^{N(A)} W(n)dn + W_{min}\right) = \xi\left(\underline{U} + \mathbb{E}v(N(A))\right),$$

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The contracting problem: solution

With linear utility, the optimal contract maximizes total surplus

$$\max_{W(\cdot),W_{min},N(\cdot)} \mathbb{E}\left[AF(N(A))\right] - \mathbb{E}\left[\int_0^{N(A)} W(n)dn + W_{min}\right]$$

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The contracting problem: solution

With linear utility, the optimal contract maximizes total surplus

$$\max_{W(\cdot),W_{min},N(\cdot)} \mathbb{E}\left[AF(N(A))\right] - \xi\mathbb{E}v(N(A))$$

$$\text{s.t. } \mathbb{E}\left(\int_0^{N(A)} W(n)dn + W_{min}\right) = \xi\left(\underline{U} + \mathbb{E}v(N(A))\right),$$

$$W(N(A)) = AF'(N(A)).$$

Solution:

1. the objective is maximized at $AF'(N(A)) = \xi v'(N(A))$
2. the incentive compatibility constraint is satisfied by $W(N) = \xi v'(N)$
3. the participation constraint is satisfied by choosing the right W_{min}

Properties of the optimal contract

- With linear utility, the optimal contract implements "first best", the condition $AF'(N) = \xi v'(N)$ maximizes total surplus
- Efficiency property dictates slope of marginal wage, base wage W_{min} adjusts to make worker agree to the contract
  - $\Rightarrow$ same response of hours as in a spot market, independent of reservation utility $\underline{U}$
- Different bargaining protocols may affect W_{min}, but not the efficiency property
  - Same contract, up to W_{min}, regardless of whether the firm, the worker, or a union specifies the wage contract.

General equilibrium

General equilibrium: overview

- Now: take our partial-equilibrium model and embed it in general equilibrium
- Key assumptions:
  - Complete asset markets
  - King-Plosser-Rebelo (1988) (KPR) preferences
- With complete markets, individual marginal utility of consumption depends *only* on aggregate consumption.
- Main results:
  - In response to anticipated changes in productivity, hours worked stay constant
    - Rigid contracts preserve balanced-growth property of KPR preferences: income and substitution effects offset
  - In response to unanticipated changes in productivity, there is no income effect: large response in hours worked

Environment

- Still two periods: 1) contracting period, 2) production period
- A continuum of firms; a continuum of workers; one-to-one matching
- Match production function: $A \times A_i \times N^{1-\alpha}$
  - Firm-level productivity $A_i \sim G$, aggregate productivity A (constant)
- Firms are owned by workers through a diversified mutual fund
- Free entry of firms: zero profits in expectation
- Each worker has separable KPR preferences,

$$U(C_i, N_i) = \log C_i - \kappa \frac{N_i^{1+\psi}}{1+\psi}.$$

- Workers can trade a complete set of state-contingent Arrow-Debreu securities

Implications for the contracting problem

- Complete markets:
  1. Worker behaves *as if* belonging to a representative family with the same preferences and $C_i = C$
  2. Worker marginal utility of consumption, $1/C_i$, is independent of firm-level shocks
  3. In the contract-negotiation stage, the worker has preferences $\frac{W_i^s}{\xi} - v(N_i)$ where $\xi = C$
- That is: contracting problem exactly the same as previously considered
- Free entry: reservation utility $\bar{U}$ adjusts so that expected profits $= 0$
- General equilibrium: $C = Y$ Equilibrium definition

Experiments

- Although static, this environment can be used to characterize dynamic responses of hours and wages to changes in aggregate productivity
- Long-run response: the response to fully anticipated changes in productivity
- Short-run response: the response to fully unanticipated changes in productivity ("MIT" shocks)

The response of hours to productivity changes

Proposition

(Balanced growth) In response to a change in aggregate productivity from A to A' that is anticipated in the contracting period, total hours are unchanged, and output moves one-for-one with productivity,

$$Y' = A'Y,$$
$$N' = N.$$

Proposition

(MIT shock) In response to an aggregate productivity shock from A to A' that is unexpected at the contracting stage, total hours and total output respond by

$$Y' = (A')^{1+(1-\alpha)/(\alpha+\psi)}\,Y,$$
$$N' = (A')^{1/(\alpha+\psi)}\,N.$$

Comparisons with other models

Comparison with other models

- To understand short-run response, compare the labor market equilibrium in our model to three comparison models:
  1. a neoclassical spot market for labor
  2. a neoclassical spot market for labor with GHH preferences
  3. rigid wages

Neoclassical spot market for labor

- Consider a competitive spot labor market with the same preferences and technology
- Labor demand is given by $W = (1-\alpha)AN^{-\alpha}$. In logs,

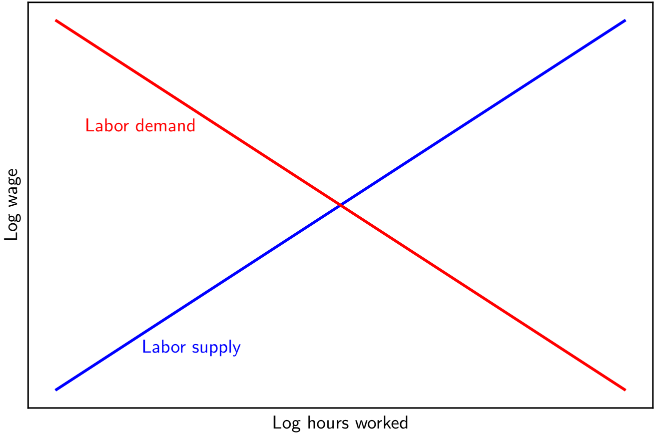
$$w = \log(1-\alpha) + a - \alpha n$$

- Labor supply is given by $\frac{W}{C} = \kappa N^{\psi}$. In logs,

$$w = \log \kappa + c + \psi n$$

- How does n respond to a?

Marshallian cross of neoclassical spot market



Labor demand: $w = \log(1 - \alpha) + a - \alpha n$

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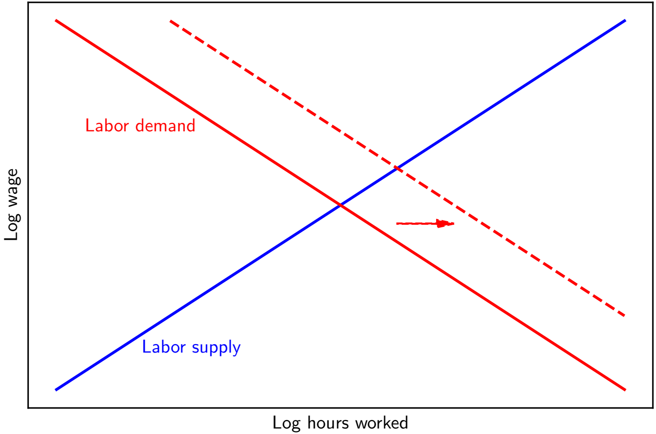

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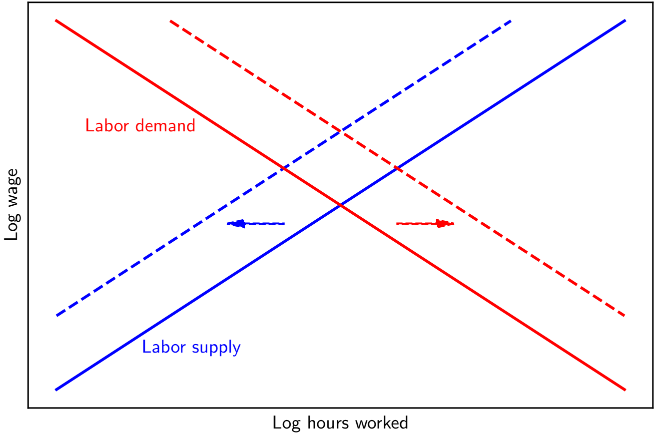
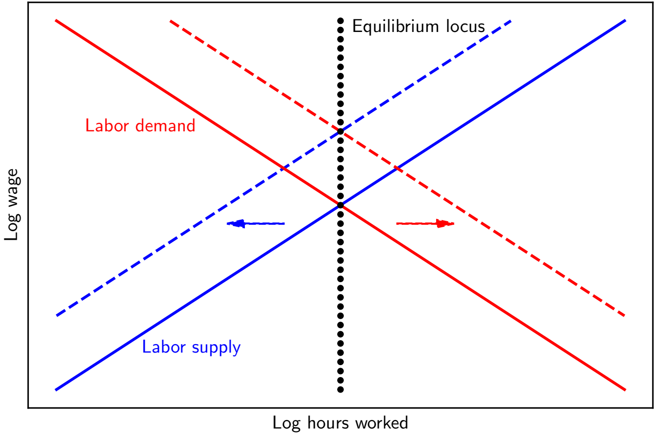


Labor demand: $w = \log(1-\alpha) + a - \alpha n$

Labor supply: $w = \log\kappa + c + \psi n$ Equilibrium: $c = a + (1-\alpha)n$

Marshallian cross of neoclassical spot market



Labor demand: $w = \log(1-\alpha) + a - \alpha n$

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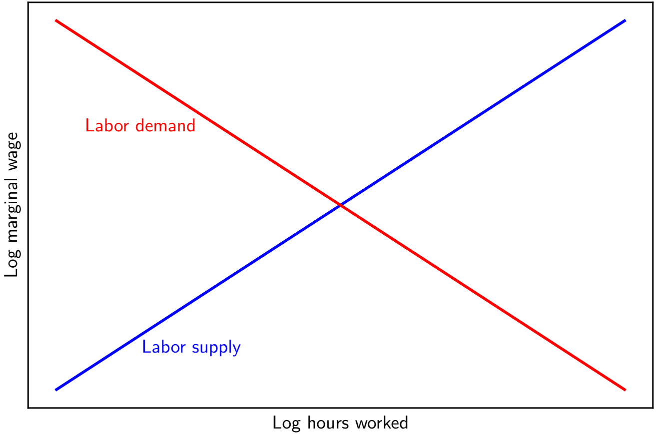
Marshallian cross with rigid wage contracts



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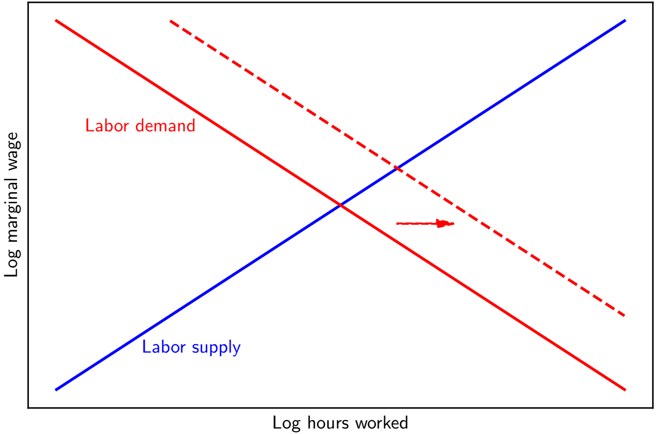

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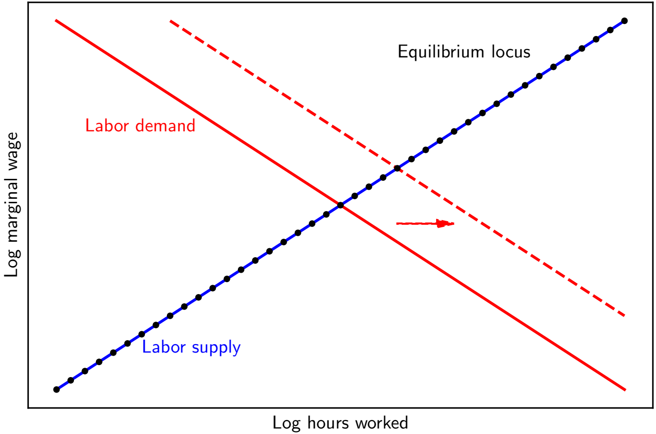
Marshallian cross with rigid wage contracts



Labor demand: $w = \log(1 - \alpha) + a - \alpha n$

Labor supply: $w = \log \kappa + c + \psi n$ Equilibrium: $c = \log \xi$

Comparison with GHH preferences

Greenwood-Hurcowitz-Huffman (1988) preferences:

$$U(C,N) = \frac{1}{1-\gamma}\left(C - \kappa\frac{N^{1+\psi}}{1+\psi}\right)^{1-\gamma} \tag{1}$$

Optimality condition

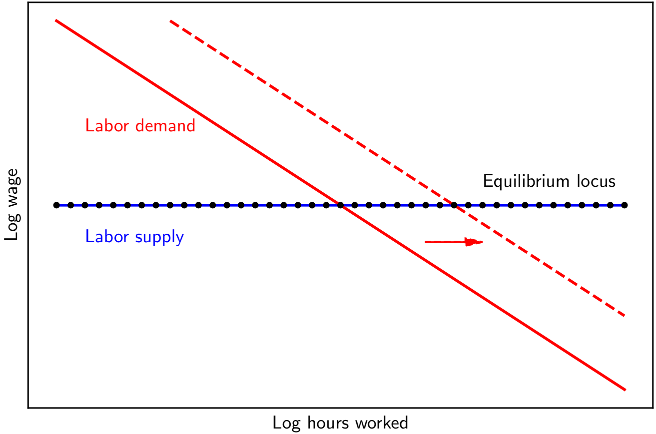
$$W = \kappa N^{\psi}$$

- Generates more plausible (stronger) hours response to aggregate shocks,
- therefore widely used in applied quant-macro literature, but...
- ... not consistent with balanced growth.

Proposition

The response of output and hours to an unexpected shock to aggregate productivity in our rigid-contracts model is identical to that in an alternative environment where hours worked are determined in a competitive spot market but where worker preferences are given by (1).

Marshallian cross with rigid wages (as in EHL)



Labor demand: $w = \log(1-\alpha) + a - \alpha n$

Labor supply: $w = \bar{w}$ i.e, as if rigid wage contract with $\psi = \infty$

Wage cyclicality

- In our model, the marginal wage is the allocative price.
- How does the average wage respond to a productivity shock?

Proposition

In response to an unexpected productivity shock, the equilibrium elasticity of the average wage with respect to hours, $\epsilon_N^{\bar{W}}$, is given by

$$\epsilon_N^{\bar{W}} = \frac{1-\alpha}{LS} - 1$$

where $LS = \frac{W^s}{Y}$ is the steady-state labor share of income.

In a standard neoclassical model with capital, $LS = 1 - \alpha$.

Dynamic model

Dynamic model

- Now: embed wage contracts in a dynamic equilibrium environment with price-level shocks and renegotiation à la Calvo (1983)
- Solve by log-linearization with respect to aggregate variables
  - We consider a perfect-foresight path to aggregate shocks (certainty equivalence holds up to a first order)
  - Underlying assumption: firm-level shocks are large relative to aggregate shocks
- Main results:
  - Labor-market equilibrium characterized by no income effect in the short run, balanced growth in the long run.
  - Frequency of resetting the contracts determines speed of transition to balanced growth. Similar to Jaimovich-Rebelo (2009) preferences
  - Phillips curve similar to Erceg-Henderson-Levin (2000), isomorphic if Frisch elasticity = ∞, but
    - No monopolistic competition
    - Workers do not 'set the wage'
    - 'Slavery concern' (Huo-{Ríos-Rull}, 2020) mitigated

Dynamic model

- Infinite horizon. Contracts reset with probability $1-\theta$. Shocks to aggregate productivity A_t and price level P_t. Let W be the *nominal* marginal wage.

- To a first order, the optimal nominal wage schedule of a particular vintage t is given by

$$W(N_{i,t+k}) = \underbrace{(1+\hat{\xi}_t)}_{\text{"allocative wage"}} \xi_{ss}\kappa N_{i,t+k}^{\psi}$$

where

$$\hat{\xi}_t = -(1-\beta\theta)\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\theta)^k(\underbrace{\hat{\lambda}_{t+k}}_{\text{m.u. of } c} - \underbrace{\hat{p}_{t+k}}_{\text{price level}})\right]$$

is the (log deviation of) average inverse marginal utility of a dollar for the duration of the contract.

Dynamic model

Proposition

Taking goods-price inflation π_t, marginal consumption utility λ_t and the initial real average allocative wage $\hat{\omega}_{-1}^{all}$ as given, the labor-market equilibrium $\{\hat{n}_t, \hat{\omega}_t^{all}\}$ is summarized by labor demand,

$$y_t = a_t + \frac{1-\alpha}{\alpha+\psi}(a_t - \omega_t^{all}), \tag{2}$$

$$n_t = \frac{1}{\alpha+\psi}(a_t - \omega_t^{all}), \tag{3}$$

a wage Phillips curve,

$$\pi_t^{all} = \beta \mathbb{E}_t \pi_{t+1}^{all} + \frac{(1-\theta)(1-\beta\theta)}{\theta}(a_t - \omega_t^{all}), \tag{4}$$

and an accounting equation,

$$\Delta\omega_t^{all} = \pi_t^{all} - \pi_t. \tag{5}$$

Comparisons with other models

Comparison I: Jaimovich-Rebelo 2009 preferences

Jaimovich and Rebelo (2009) considered a neoclassical spot labor market in which workers have a per-period utility function:

$$U(C_t, N_t, X_t) = \frac{\left(C_t - \frac{\kappa N_t^{1+\psi} X_t}{1+\psi}\right)^{1-\sigma} - 1}{1-\sigma} \tag{6}$$

where $X_{i,t}$ represents a habit, depending on past consumption.

Three desirable properties of Jaimovich-Rebelo 2009 preferences:

- limited income effects in the short run
- balanced growth in the long run
- a parameter (habit persistence) that controls the speed of convergence to balanced growth

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where $X_{i,t}$ represents a habit, depending on past consumption.

The log-linearized labor-supply condition from these preferences is

$$\hat{x}_t + \psi \hat{n}_t = \hat{a}_t - \alpha \hat{n}_t.$$

Compare with the labor-demand condition from our model,

$$\hat{\omega}_t^{all} + \psi \hat{n}_t = \hat{a}_t - \alpha \hat{n}_t.$$

The sluggishness of allocative wages in our model play the same role as habits in Jaimovich and Rebelo (2009).

Comparison II: Erceg-Henderson-Levin (2000) wage rigidity

EHL: workers are in monopolistic competition, set their own wage ex ante, and are required to supply whatever hours demanded ex post.

The resulting labor-market equilibrium is given by

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w - \gamma^{EHL}(\hat{a}_t + \hat{\lambda}_t - (\alpha + \psi)\hat{n}_t)),$$
$$\Delta \hat{a}_t - \alpha \Delta \hat{n}_t = \pi_t^w - \pi_t.$$

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$$\Delta \hat{a}_t - \alpha \Delta \hat{n}_t = \pi_t^w - \pi_t.$$

By comparison, our model labor-market equilibrium is given by

$$\pi_t^{w^{all}} = \beta \mathbb{E}_t \pi_{t+1}^{w^{all}} - \gamma(\hat{a}_t + \hat{\lambda}_t - (\alpha + \psi)\hat{n}_t)),$$
$$\Delta \hat{a}_t - (\alpha + \psi)\Delta \hat{n}_t = \pi_t^{w^{all}} - \pi_t.$$

Replacing α with $\alpha + \psi$ is key for quantification (upward sloping supply curve instead of horizontal).

The mechanics of new-Keynesian models

- We thus have a new-Keynesian model without monopolistic competition (markups, profits,...). No one "sets the wage"
- What is essential for the new-Keynesian paradigm?
  - *Contracts* are nominally rigid
  - In the context of goods prices, it may be natural to think of these contracts as "prices", less so for wage contracts
    - How these contracts are formed is not essential. Unions, workers, firms, government,...
  - Output is *demand determined* (in the labor market, the firm has the 'right to manage')

Conclusion/going forward

Conclusion

- Introduced a framework of rigid wage contracts with core assumptions:
  1. firms have right to manage
  2. contracts are rigid (cannot be conditioned on shocks; cannot be renegotiated with certainty)
- Model purposefully simple in all other dimensions (separable preferences, spot market for contracts, complete asset markets etc.)
- Key implication: rigid wage contracts mutes wealth effects on hours worked - hours worked *as if* spot labor market with GHH/JR preferences
- Also,
  1. generate novel predictions for wage dynamics
  2. provide a foundation for a new Keynesian Phillips curve
- Our framework is 'plug and play' in quantitative business-cycle models

Going forward

- Past: establish theoretical benchmark
- Future:
  1. study quantitative implications of adding realistic frictions
  2. confront theory with data
- One avenue: how do incomplete asset markets affect shape of wage contracts?
  - Motivated by the vast literature documenting that incomplete asset markets fundamentally change business cycle dynamics
- Other topics:
  - Use framework together with frictional labor markets to study interplay of extensive and intensive variations in hours worked
  - Confront model with data (and vice versa)

Rigid wage contracts with incomplete asset markets

Rigid wage contracts with incomplete asset markets

- Goal: study the micro- and macroeconomic implications of optimal rigid contracts between risk-neutral firms and risk-averse workers that can only save in risk-free assets as in Aiyagari (1994)

- Sharp results under complete asset markets

Rigid wage contracts with incomplete asset markets

- Goal: study the micro- and macroeconomic implications of optimal rigid contracts between risk-neutral firms and risk-averse workers that can only save in risk-free assets as in Aiyagari (1994)
- Sharp results under complete asset markets
- New: also sharp results under complete financial autarky
  - need to solve for the optimal contract numerically
  - optimal contract can be summarized by two sufficient statistics

Dynamic model under financial autarky

Proposition

The labor-market equilibrium for a continuum of workers under financial autarky, given paths for aggregate productivity a_t and inflation π_t, is given by

$$
\begin{aligned}
y_t &= a_t + (1-\alpha)\varepsilon_Y(a_t - \omega_t^{all}), \\
n_t &= \varepsilon_N(a_t - \omega_t^{all}), \\
\pi_t^{all} &= \beta \mathbb{E}_t \pi_{t+1}^{all} + \frac{(1-\theta)(1-\beta\theta)}{\theta}(a_t - \omega_t^{all}), \\
\Delta\omega_t^{all} &= \pi_t^{all} - \pi_t,
\end{aligned}
$$

where ε_Y and ε_N are derived from the optimal wage contract.

The elasticities ε_Y and ε_N can be computed from numerically solving for the optimal contract.

Same equations as under complete markets. Under complete markets, $\varepsilon_Y = \varepsilon_N = \frac{1}{\alpha+\psi}$.

Extra slides

Equilibrium definition

Definition

A competitive equilibrium consists of a wage-hours schedule $W^s(N)$, an hours schedule $N(A_i)$, consumption C, and aggregate production Y such that

- given the worker's inverse marginal utility of consumption ξ, $W^s(N)$ solves the contracting problem,
- the reservation utility $\underline{U}$ is such that $\mathbb{E}\left[A_i F(N_i) - W^s(N_i)\right] = 0$,
- ex-post hours for worker i, $N_i = N(A_i)$, satisfy firm optimality given the contract $W^s(N_i)$ and realized productivity A_i,
- the goods market clears: $C = Y$ with $Y = \int_{i=0}^{1} A_i F(N_i) di$,
- and the inverse marginal utility of consumption is $\xi = \frac{1}{u'(C)} = C$.

Back

Characterization of equilibrium contract under financial autarky

Proposition

Let $H(Z, \Pi, \Pi') = f_Z(Z)\left\{[u(Z\Pi' - \Pi) - v(F^{-1}(\Pi')] + \lambda\Pi\right\}$. *The optimal wage contract is characterized by:*

$$\lambda = \mathbb{E}[u'(C)],$$

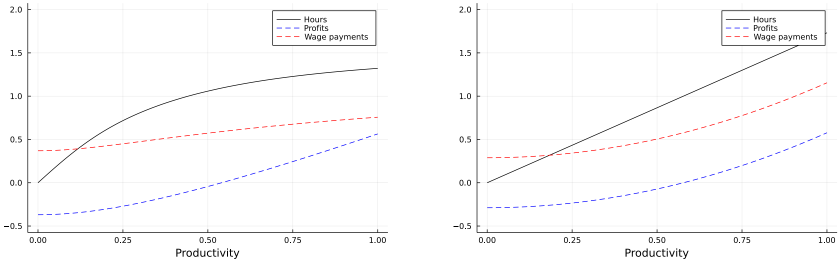
$$\mathbb{E}[\Pi(Z)] = 0,$$

$$\frac{\partial H}{\partial \Pi} = \frac{d}{dZ}\frac{\partial H}{\partial \Pi'},$$

$$\Pi'(0) = 0.$$

(Euler-Lagrange equation)

Equilibrium contracts: a numerical example



(a) Hours, profits and wage payments as a function of productivity for the optimal contract under financial autarky.

(b) Hours, profits and wage payments as a function of productivity for the optimal contract under complete markets.

Figure 1: Equilibrium outcomes under (a) financial autarky and (b) complete markets.

I use the following functional forms: $u(C) = \log C$, $v(N) = N^2/2$, $F(N) = N$ and $A \sim \text{Unif}(0, 1)$. Under financial autarky, there is an additional insurance motive.

Equilibrium contracts: a numerical example

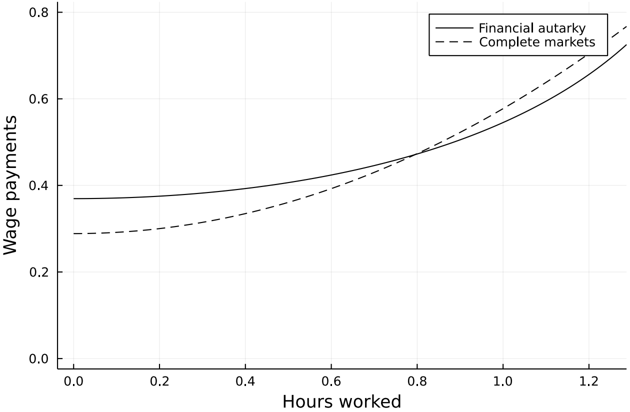


Figure 2: The equilibrium contract under financial autarky (solid) and complete markets (dashed).

The dynamic contracting problem reduces to the static contracting problem

Proposition

With balanced-growth preferences, $u(C) = \log(C)$, the optimal contract for the dynamic contracting problem is given by $\Pi(Z) = \bar{P}\bar{A}\tilde{\Pi}(Z/(\bar{P}\bar{A})$ where $\tilde{\Pi}$ is the solution to the static contracting problem.

- Intermediate result: under balanced-growth preferences, the optimal contract scales with productivity.
- General result: the optimal contract scales with the numeraire.

Although we need to turn to the computer to characterize the optimal contract, we can analytically characterize the first-order perturbations of the dynamic contract!

The optimal contract scales with $\xi_0 = (1 - \beta\theta)\mathbb{E}_0 \sum_{t=0}^{\infty}(\beta\theta)^t(a_t + p_t)$.